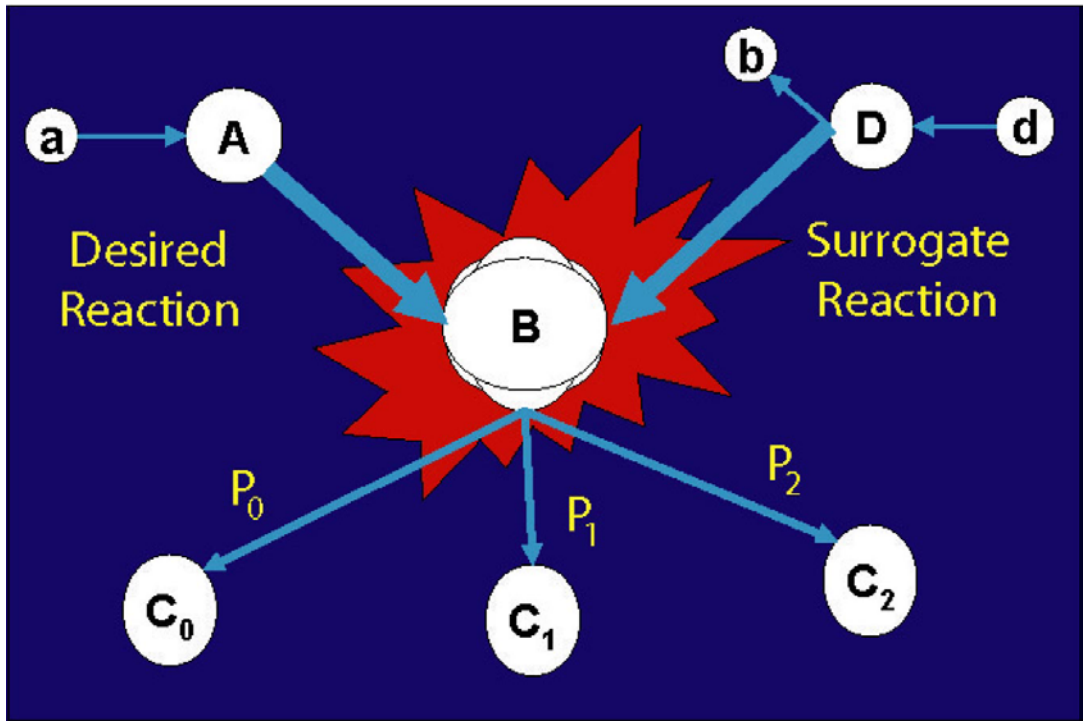
Indirect Approaches

to determine ANCs with application to capture reactions in nuclear astrophysics



R. Tribble

Texas A&M University

January, 2004

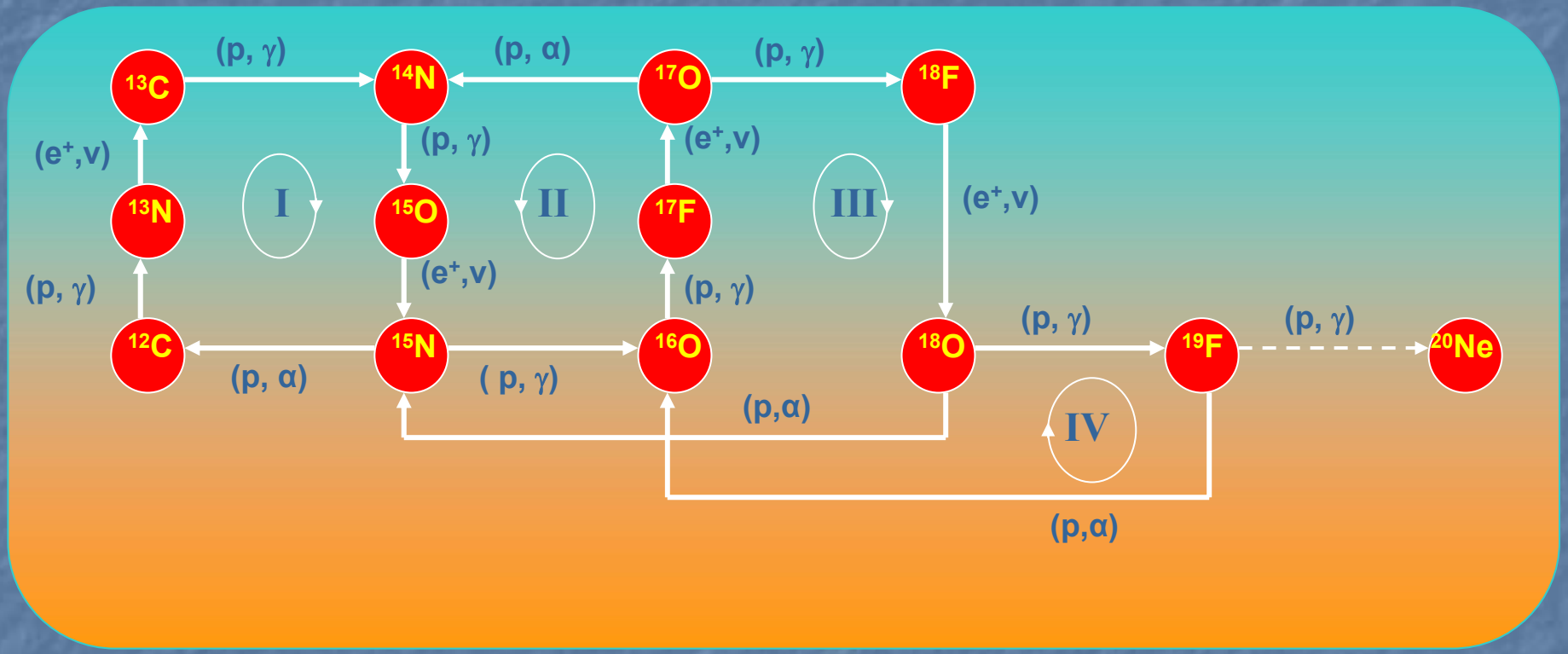
Outline

- Nuclear Astrophysics
- Reaction Rates and ANCs
- ANCs via Transfer Reactions
- Test cases
- Recent Results
- ANCs via Breakup Reactions
- Summary and Outlook

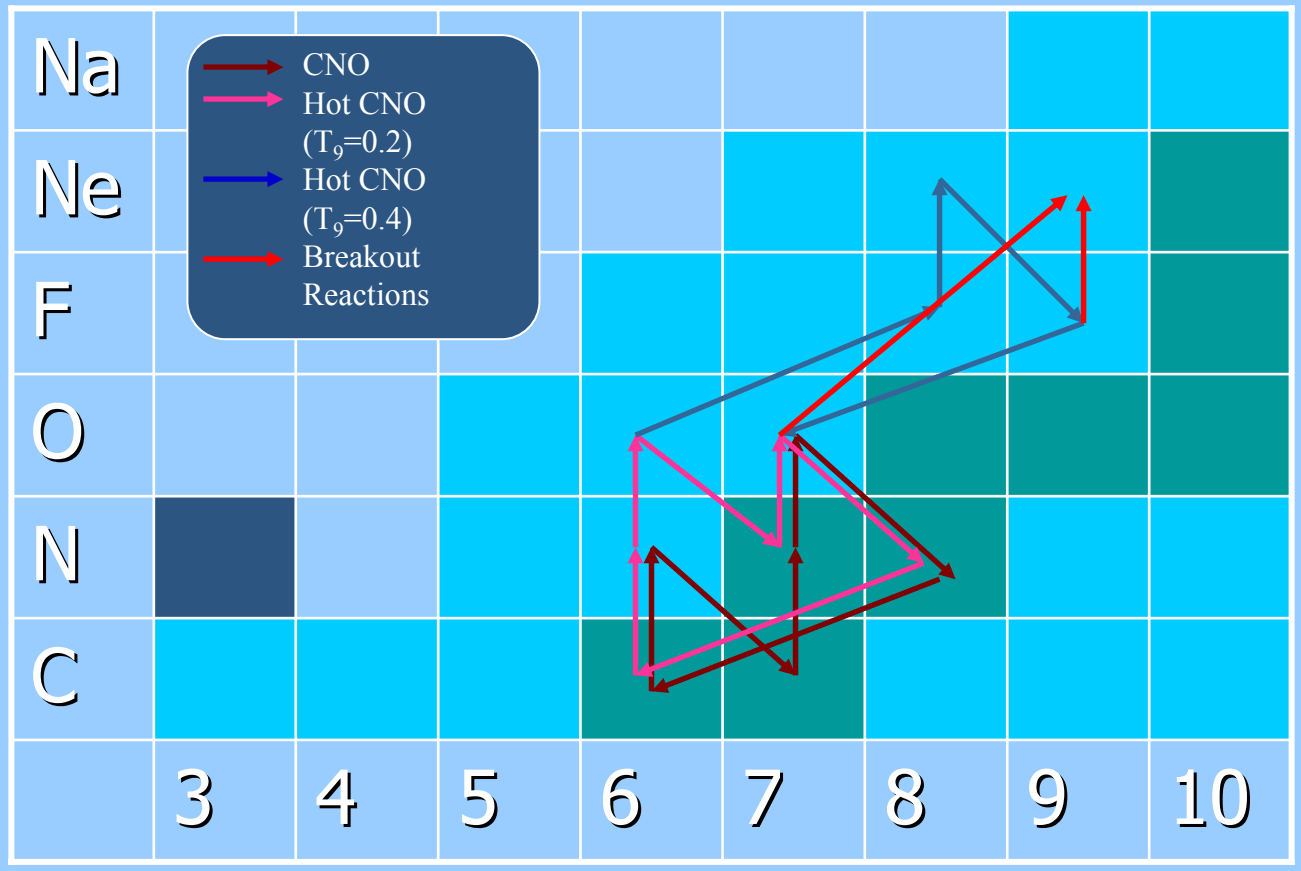
Nuclear Astrophysics Issues

- **Stellar evolution** – *pp* chains, CNO, …
- **Explosive burning** – rα*p*, HCNO, …
- Need **reaction rates** and **β-decay τ's**
- **(p,γ)** [(α,γ)] capture reactions $\Rightarrow$ **ANC**s
  - resonant and direct contributions

CNO Cycles

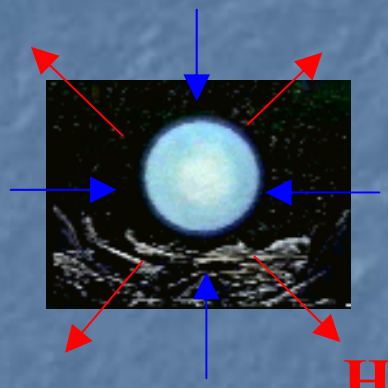

^{13}C
^{14}N
^{17}O
^{18}F
^{13}N
^{15}O
^{17}F
^{12}C
^{15}N
^{16}O
^{18}O
^{19}F
^{20}Ne
(p, γ)
(p, α)
(p, γ)
(e+,v)
(p, γ)
(e+,v)
(e+,v)
(e+,v)
(p, γ)
(p, γ)
(p, α)
(p, γ)
(p,α)
(p, γ)
(p, γ)
(p,α)
I
II
III
IV

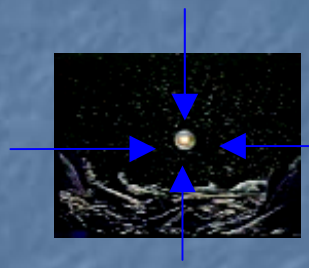
Hot CNO Cycle and $^{13}N(p,\gamma)^{14}O$



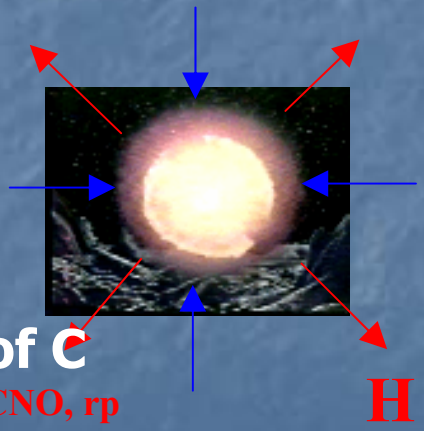
http://csep10.phys.utk.edu/guidry/NC-State-html/cno.html

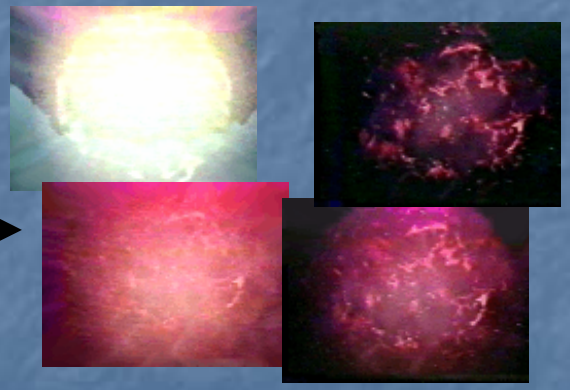
Fate of Massive Pop III Stars





3α





3α->sufficient ^{12}C??

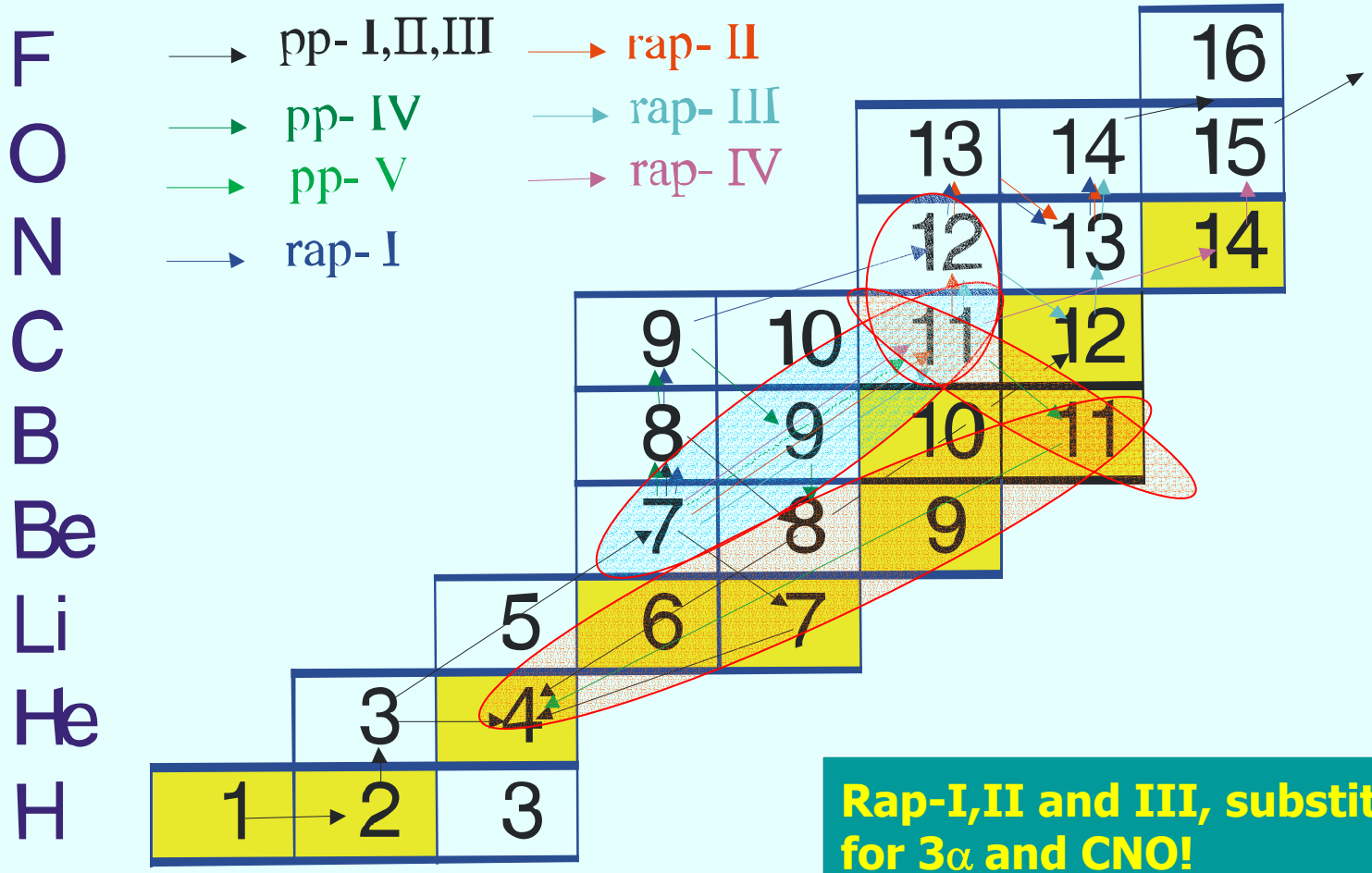
Critical Mass Fraction of C

1E-9 (A. Heger et al.)

1E-10 (Weiss et al. 2000)

1E-12 (Siess et al. 2002)

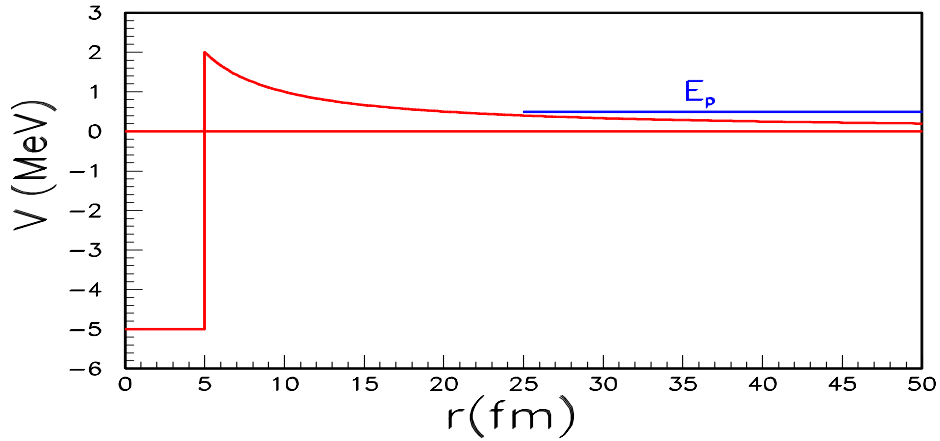
Updated Reaction Sequences in Pop III Stars



Rap-I,II and III, substitution for 3α and CNO!
(Wiescher et al., 1989)

Radiative *p* or α Capture

- Classical barrier penetration problem!



- Low energies ⇒ capture at large radii
- VERY small cross sections ⇒ define **S** factor

$$\sigma(E) = \frac{S(E)}{E}\exp\{-2\pi\eta(E)\} \qquad \eta(E) = \frac{Z_1 Z_2 e^2}{\hbar v}$$

Direct Radiative proton capture

$$\sigma \propto |M|^2 \qquad [S(E) = Ee^{2\pi\eta}\sigma]$$

M is:

$$M = \left\langle \phi_A(\xi_B, \xi_p, \xi_{Bp}) \middle| \hat{O}(r_{Bp}) \middle| \phi_B(\xi_B)\phi_p(\xi_p)\psi_i^{(+)}(r_{Bp}) \right\rangle$$

Integrate over ξ:

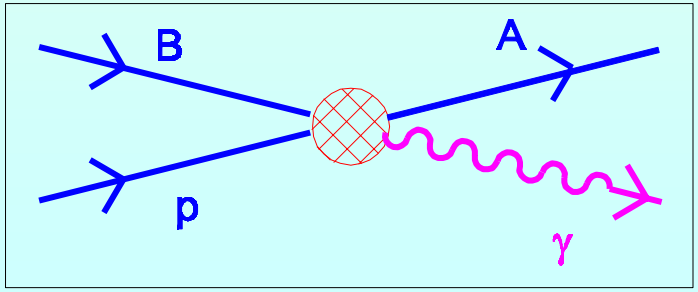
$$M = \left\langle I_{Bp}^A(r_{Bp}) \middle| \hat{O}(r_{Bp}) \middle| \psi_i^{(+)}(r_{Bp}) \right\rangle$$

Low B.E.:

$$I_{Bp}^A(r_{Bp}) \overset{r_B > R_N}{\approx} C_{Bp}^A \frac{W_{-\eta_A, l+\frac{1}{2}}(2\kappa_{Bp} r_{Bp})}{r_{Bp}}$$

Find:

$$\sigma_{capture} \propto (C_{Bp}^A)^2$$



Radiative Capture of Charged Particle

$$M_{ch}^{R} = ie^{-i\phi}\frac{\Gamma_p^{1/2}}{(E_r - E) - i\frac{\Gamma}{2}}\left(N\lambda v^{-1/2}\left\langle C_{Ap}^{B}\frac{W_{-\eta,l+\frac{1}{2}}(2k_B r)}{r}\middle\| H^L \middle\| O\right\rangle\right)$$

r>R_N

$$M_{\text{int}}^{R} = ie^{-i\phi}\frac{\Gamma_p^{1/2}}{(E_r - E) - i\frac{\Gamma}{2}}\left(\lambda\hbar^{1/2}\left\langle I_{Ap}^{B}(2k_B r)\middle\| H^L \middle\| x(r)\right\rangle\right)$$

r<R_N

$$O = F + iG$$

$$\Gamma_{\gamma 0} \equiv \left|\Gamma_{\gamma\ f}^{1/2} - (\delta\Gamma_{\gamma\ f})^{1/2}\right|^2$$

Real, ?

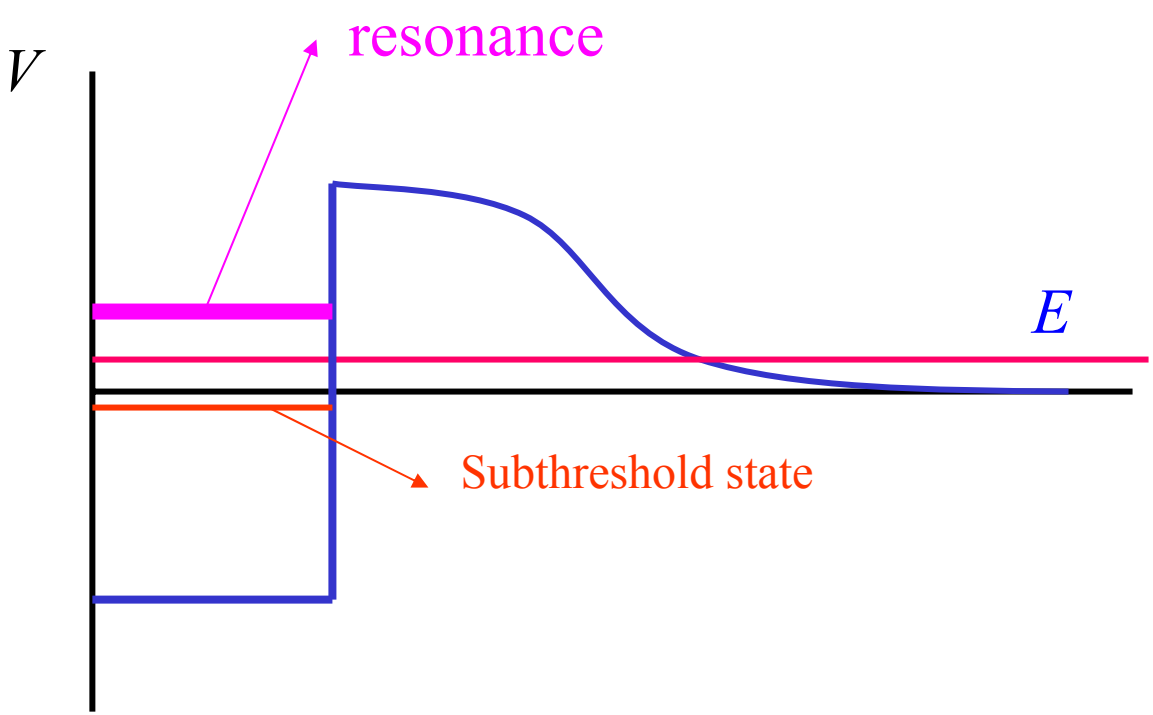
Complex, ANC

observed

$$\sigma(E) \propto \left|U_{l_i=0,j_i=2}^{(D,E1)} + U_{l_i=0,j_i=2}^{(R,E1)}\right|^2 + \left|U_{l_i=0,j_i=1}^{(D,E1)}\right| + \left|U_{cc'}^{(R,E2)}\right|^2 + \left|U_{cc'}^{(R,M1)}\right|^2$$

Capture through subthreshold states

well known problem for many years



Examples:

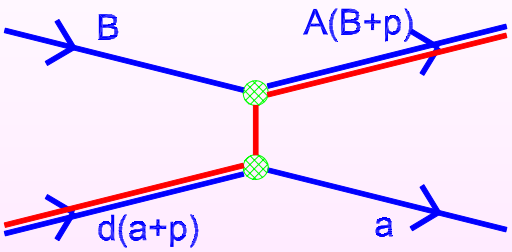
$^{12}C(\alpha,\gamma)^{16}O$

$^{13}C(\alpha,n)^{16}O$

$^{14}N(p,\gamma)^{15}O$

$^{20}Ne(p,\gamma)^{21}Na$

Transfer Reaction



Transition amplitude:

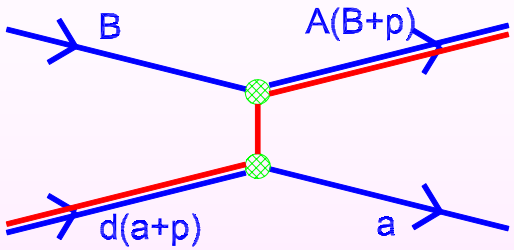
$$M = \sum \left\langle \chi_f^{(-)} I_{Bp}^{A} \middle| \Delta V \middle| I_{ap}^{d} \chi_i^{(+)} \right\rangle$$

Standard approach:

$$I_{ap}^{b} = \sum S_{ap}^{\frac{1}{2}} \phi_{n_d l_d j_d}(r_{ap})$$

$$\frac{d\sigma}{d\Omega} = \sum S_{Bpl_A j_A} S_{apl_d j_d} \sigma_{l_A j_A l_d j_d}^{DW}$$

Transfer Reaction



Transition amplitude:

$$M = \sum \left\langle \chi_f^{(-)} I_{Bp}^{A} \middle| \Delta V \middle| I_{ap}^{d} \chi_i^{(+)} \right\rangle$$

Peripheral transfer:

$$I_{Bp}^{A} \overset{r_{Bp} > R_N}{\approx} C_{Bp}^{A} \frac{W_{-\eta_A, l+\frac{1}{2}}(2\kappa_{Bp} r_{Bp})}{r_{Bp}}$$

$$\frac{d\sigma}{d\Omega} = \sum \frac{(C_{Bpl_A j_A}^{A})^2 (C_{apl_d j_d}^{d})^2}{b_{Bpl_A j_A}^2 \, b_{apl_d j_d}^2} \sigma_{l_A j_A l_d j_d}^{DW} = \sum R (C_{Bp}^{A})^2 (C_{ap}^{d})^2$$

What are the **critical** issues for determining **ANCs** from **transfer reactions**?

- **Reaction model** -
  - DWBA - one-step vs multi-step
  - optical model parameters
  - other approaches – adiabatic model?

- **Reaction 'mechanics'**
  - must verify reaction is peripheral
  - need absolute cross section
  - generally need data at very forward angles

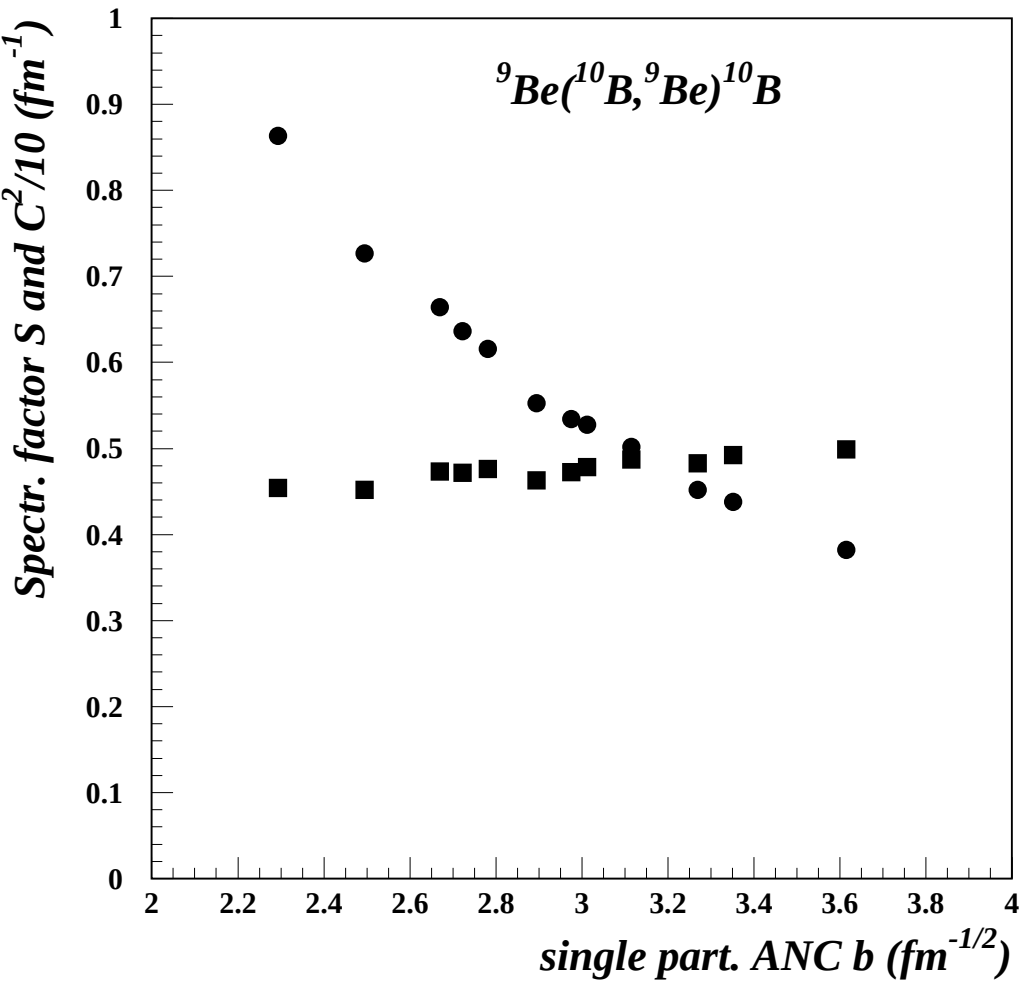
Some **ANC's** measured using stable beams

- ^{9}**Be** + ***p*** $\leftrightarrow$ ^{10}**B*** [^{9}Be(^{3}He,*d*)^{10}B;^{9}Be(^{10}B,^{9}Be)^{10}B]
- ^{7}**Li** + ***n*** $\leftrightarrow$ ^{8}**Li** [^{12}C(^{7}Li,^{8}Li)^{13}C]
- ^{12}**C** + ***p*** $\leftrightarrow$ ^{13}**N** [^{12}C(^{3}He,*d*)^{13}N]
- ^{12}**C** + ***n*** $\leftrightarrow$ ^{13}**C** [^{12}C(*d*,*p*)^{13}C]
- ^{13}**C** + ***p*** $\leftrightarrow$ ^{14}**N** [^{13}C(^{3}He,*d*)^{14}N;^{13}C(^{14}N,^{13}C)^{14}N]
- ^{14}**N** + ***p*** $\leftrightarrow$ ^{15}**O** [^{14}N(^{3}He,*d*)^{15}O]
- ^{16}**O** + ***p*** $\leftrightarrow$ ^{17}**F*** [^{16}O(^{3}He,*d*)^{17}F]
- ^{20}**Ne** + ***p*** $\leftrightarrow$ ^{21}**Na** [^{20}Ne(^{3}He,*d*)^{21}Na]

beams $\approx$ 10 MeV/u

Check for Peripheral Transfer

[i.e. is ***R*** constant as ***b*** is varied?]



- $d\sigma/d\Omega$ as function of radial cutoff
- localization of angular momentum transfer
- vary *r* and *a* in Wood-Saxon well i.e. vary *b*

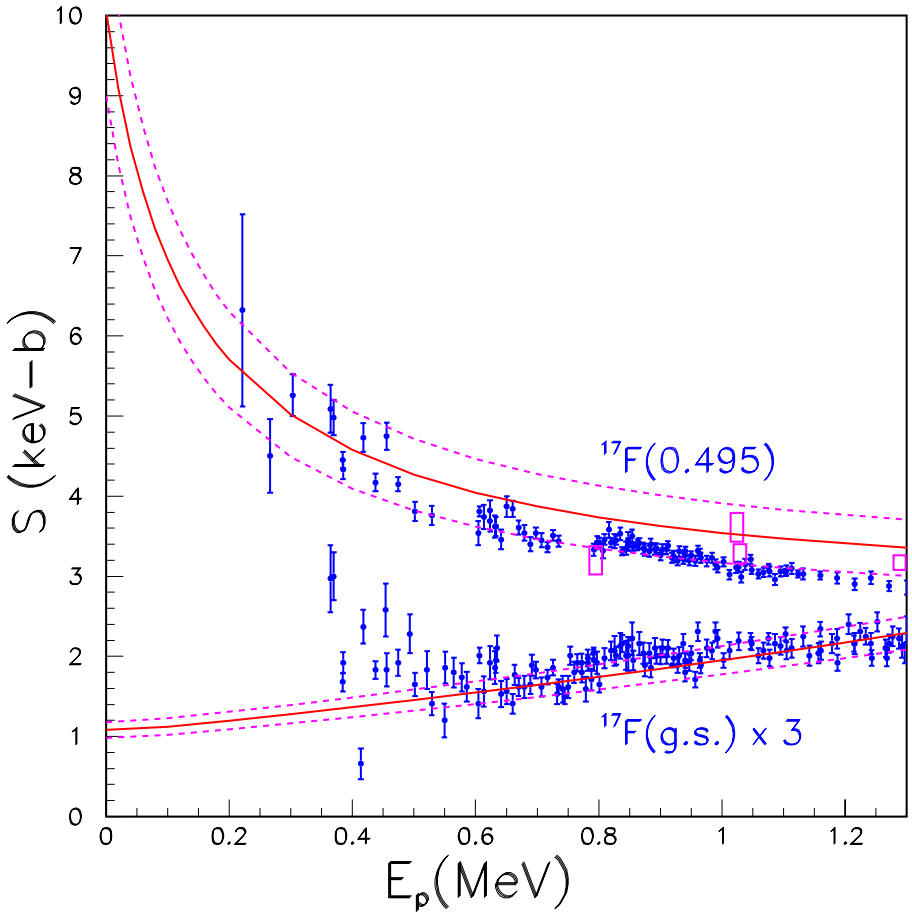
S factor for $^{16}O(p, \gamma)^{17}F$

- ANC's $\Leftarrow$ $^{16}O(^{3}He,d)^{17}F$

  $(C^2)_{gnd} = 1.08 \pm .10\ fm^{-1}$

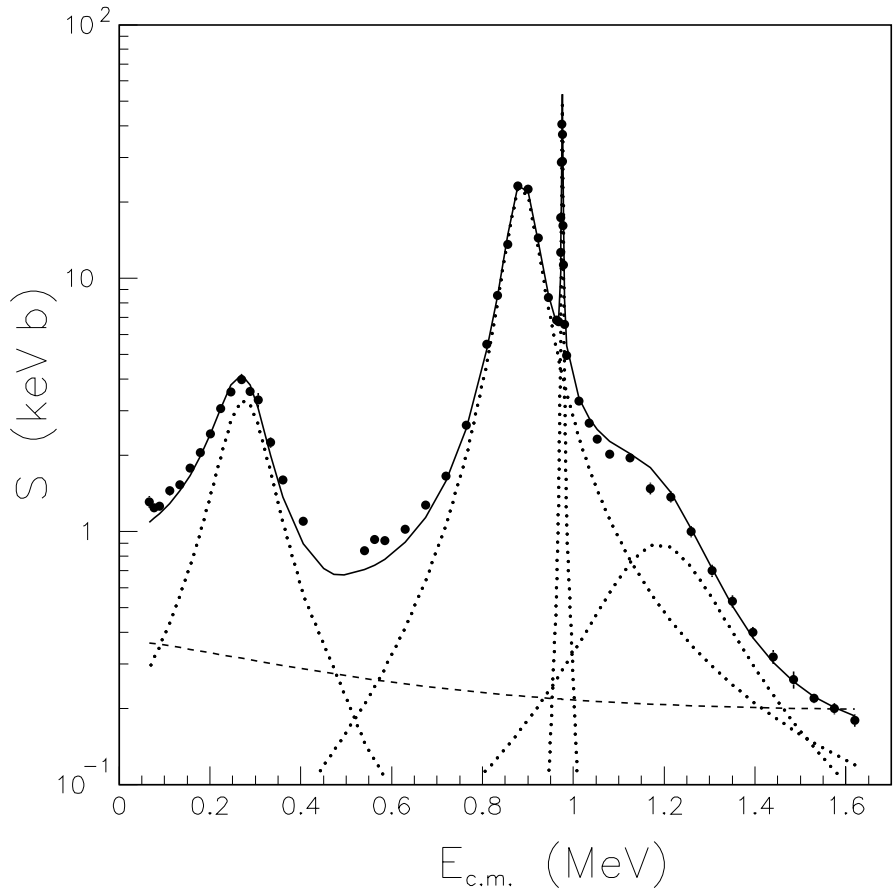
  $(C^2)_{ex} = 6490 \pm 680\ fm^{-1}$

- Direct Capture data from Morlock, et. al



S factor for $^{9}Be(p, \gamma)^{10}B$

- ANC's $\Leftarrow$ $^{9}Be(^{3}He,d)^{10}B$ and $^{9}Be(^{10}B, {}^{9}Be)^{10}B$
- R-Matrix fit to ground plus excited states (includes interference)
- Data from Zahnow, . . ., Rolfs et. al (1995)
- Uses known values for E_r and Γ_γ



S factor for $^{14}N(p, \gamma)^{15}O$

- ANC's $\Leftarrow$ $^{14}N(^{3}He,d)^{15}O$

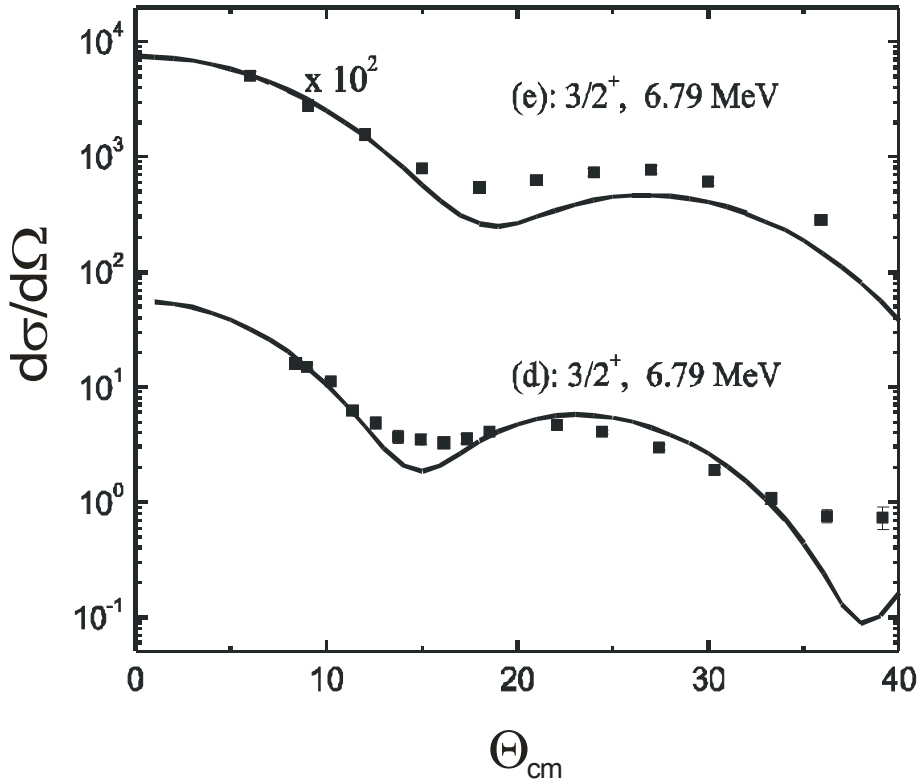
  (d) $\Rightarrow$ Rez/TAMU (27 MeV)

  (e) $\Rightarrow$ TUNL (20 MeV)

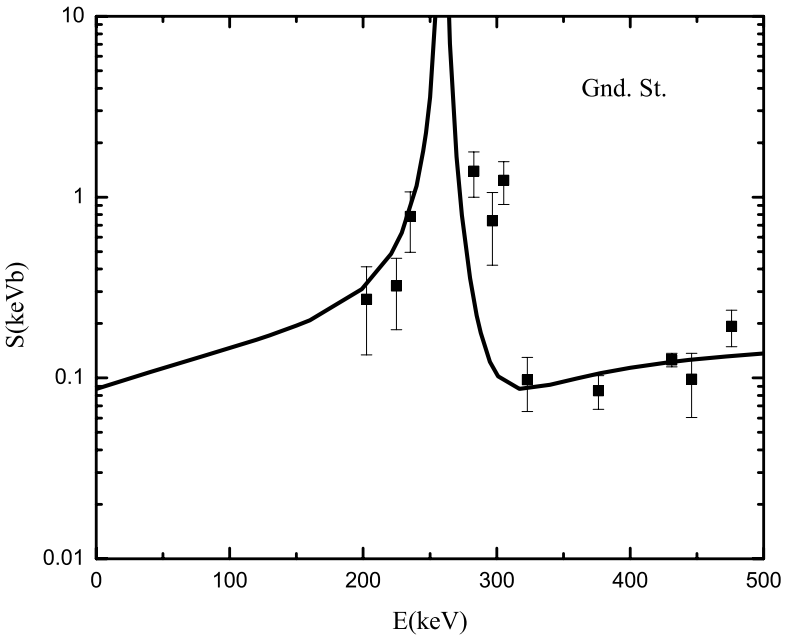
- NRC to subthreshold state at E_x = 6.79 MeV

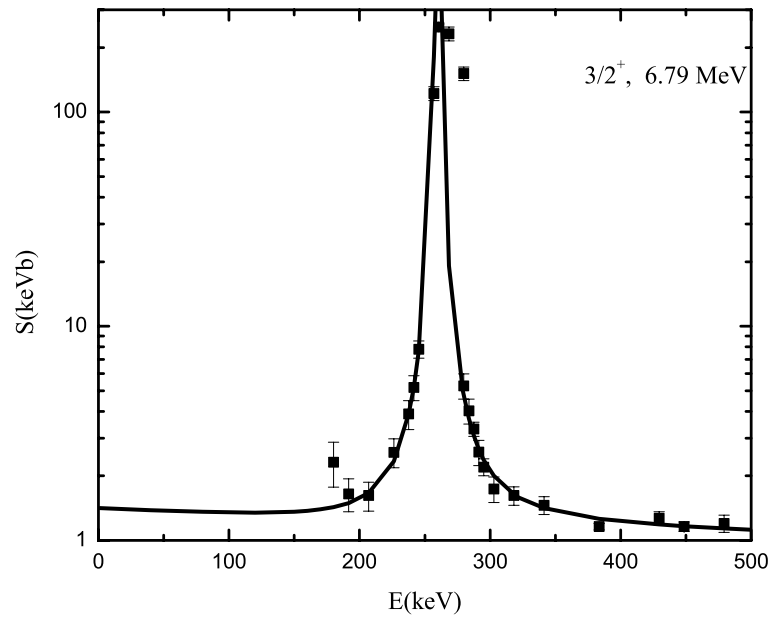
- Subthreshold resonance width from Bertone, *et al.*

- R-Matrix fits to data from Schröder, *et al.*



S factor for $^{14}N(p, \gamma)^{15}O$





- $C^2(E_x=6.79\ \text{MeV}) \approx 27\ \text{fm}^{-1}$ [non-resonant capture to this state dominates S factor]
- $S(0) = 1.40 \pm 0.20$ keV·b for $E_x = 6.79$ MeV
- $S_{tot}(0) = 1.70 \pm 0.22$ keV·b

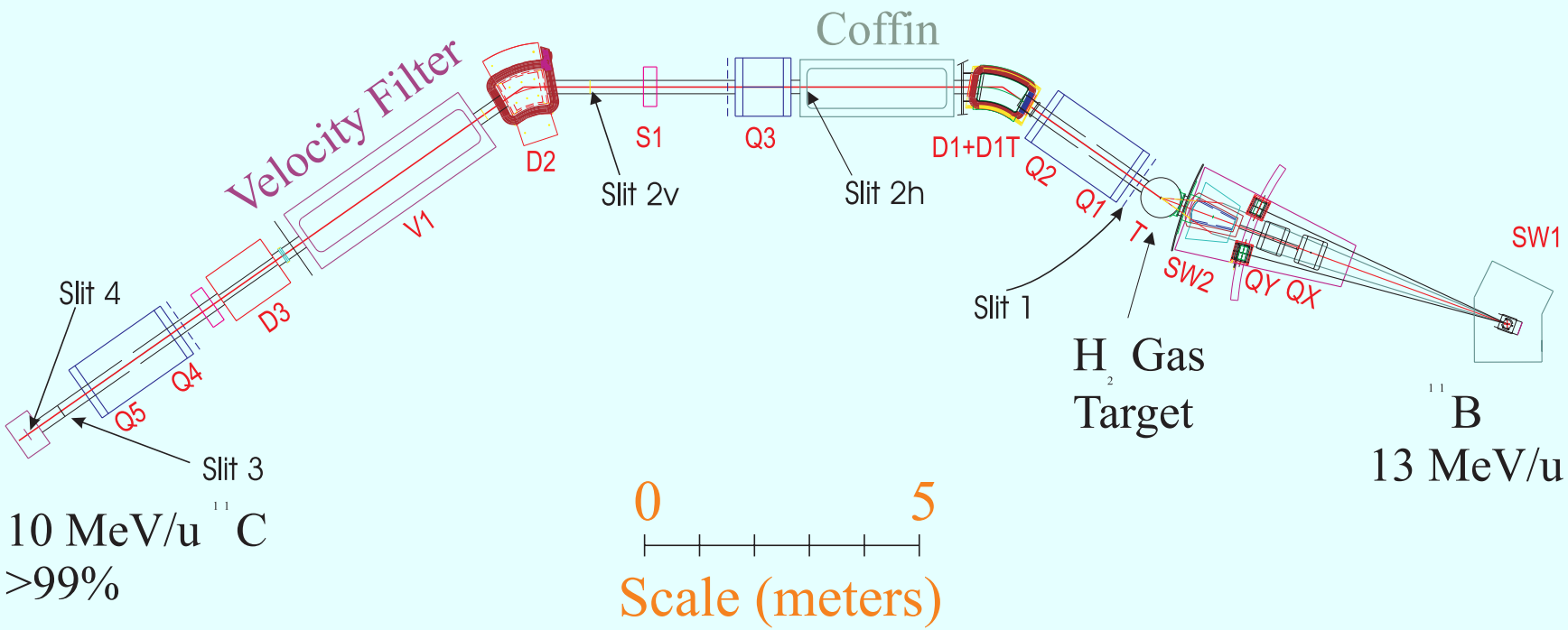
Reaction rate at .007 < T9 < 0.1 about 1/2 that of NACRE.

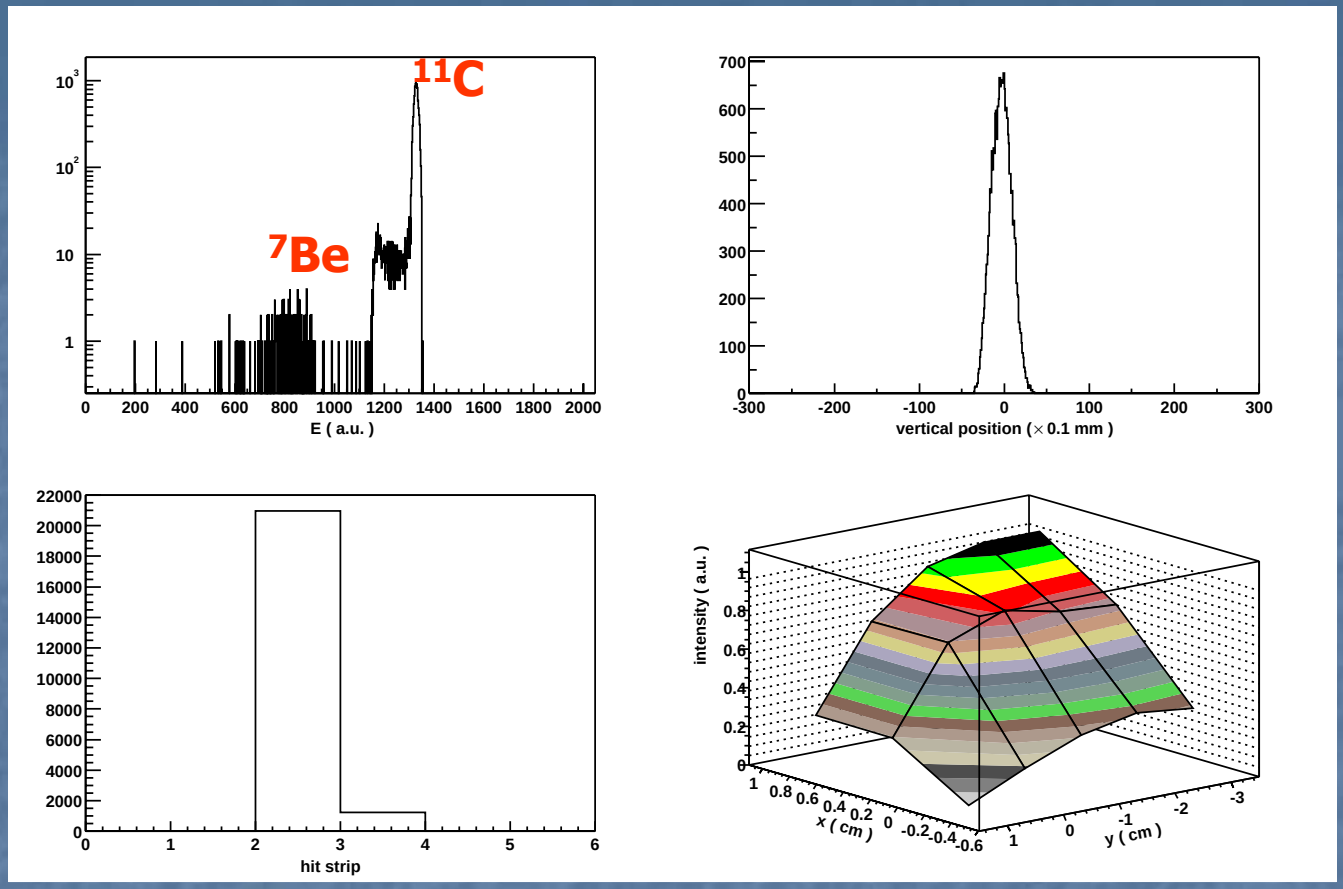
ANC's measured by radioactive (rare isotope) beams

- ^{7}Be + $p \leftrightarrow$ ^{8}B [^{10}B(^{7}Be,^{8}B)^{9}Be] [TAMU]
  [^{14}N(^{7}Be,^{8}B)^{13}C] [TAMU]
  [d(^{7}Be,^{8}B)n]
- ^{8}B + $p \leftrightarrow$ ^{9}C [d(^{8}B,^{9}C)n]
- ^{11}C + $p \leftrightarrow$ ^{12}N [^{14}N(^{11}C,^{12}N)^{13}C] [TAMU]
- ^{13}N + $p \leftrightarrow$ ^{14}O [^{14}N(^{13}N,^{14}O)^{13}C] [TAMU]
- ^{17}F + $p \leftrightarrow$ ^{18}Ne [^{14}N(^{17}F,^{18}Ne)^{13}C]
  {ORNL (TAMU collaborator)}

beams ≈ 10 - 12 MeV/u

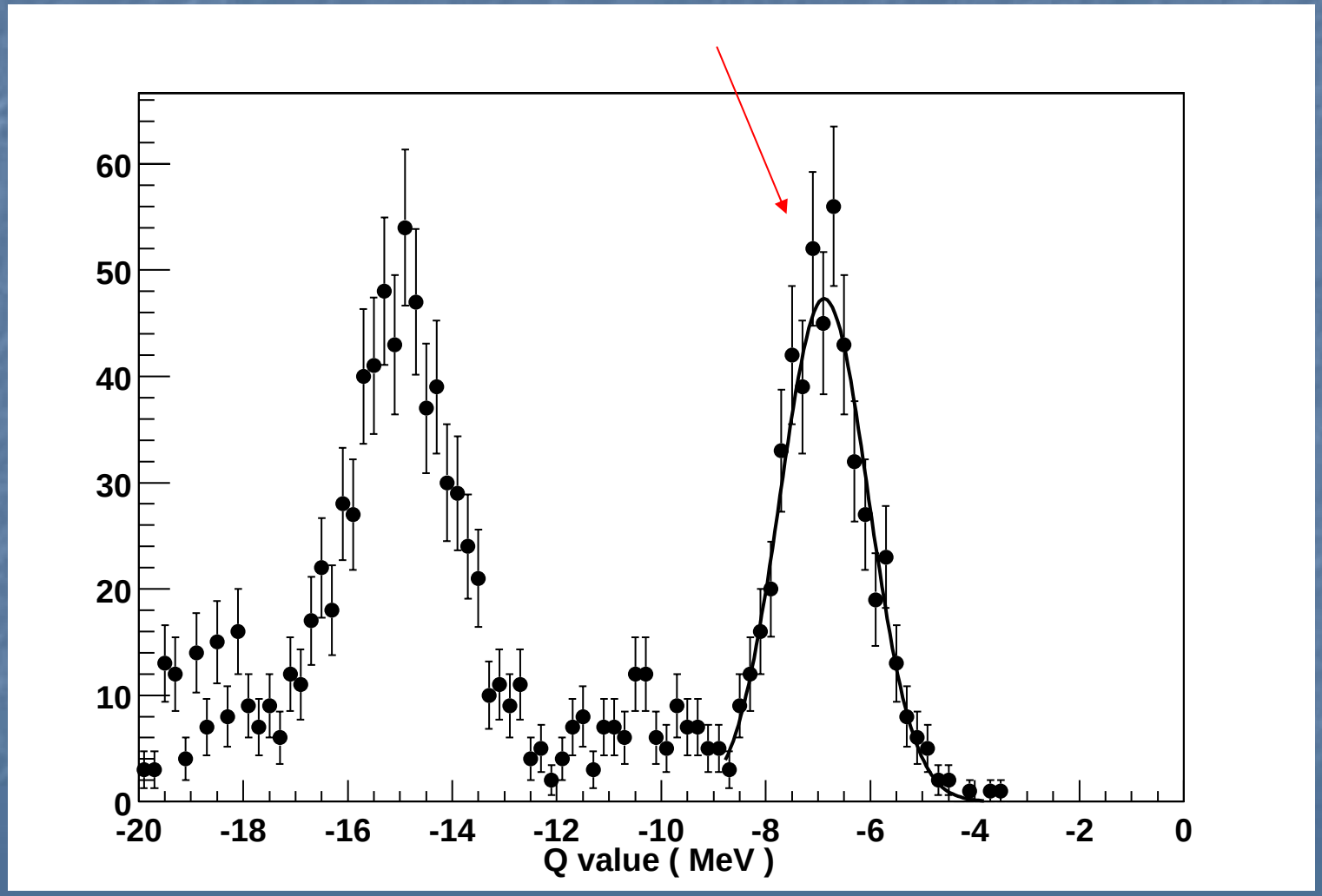
Momentum Achromat Recoil Separator



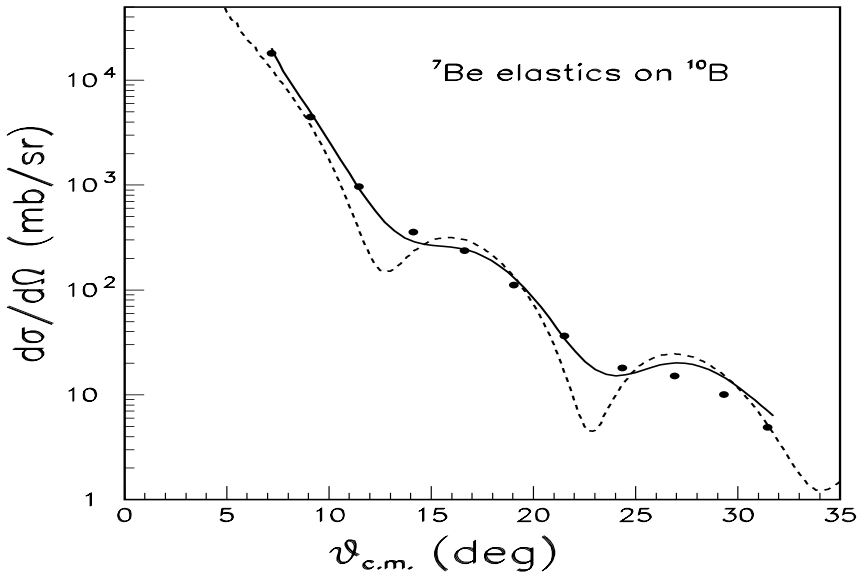


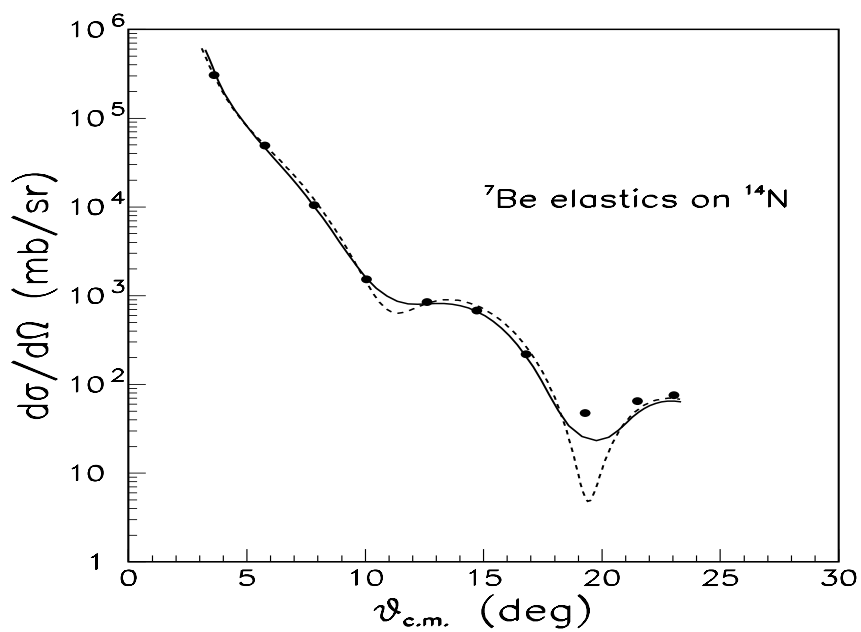
Primary Beam : $^{11}B^{2+}$@13 MeV/u, 800 enA
Primary Reaction : $^{11}B(^{1}H,n)^{11}C$
Secondary beam : ^{11}C
Intensity>400 kHz, PURITY>99%
E=110 MeV, ΔE=1.6 MeV (FWHM)
ΔX=3 mm (FWHM), ΔY=3.2 mm (FWHM)
$\Delta\theta$=1.8 deg(FW), $\Delta\phi$=1.9 deg (FW)

$^{14}N(^{11}C,^{12}N)^{13}C$



S factor for $^{7}Be(p, \gamma)^{8}B$

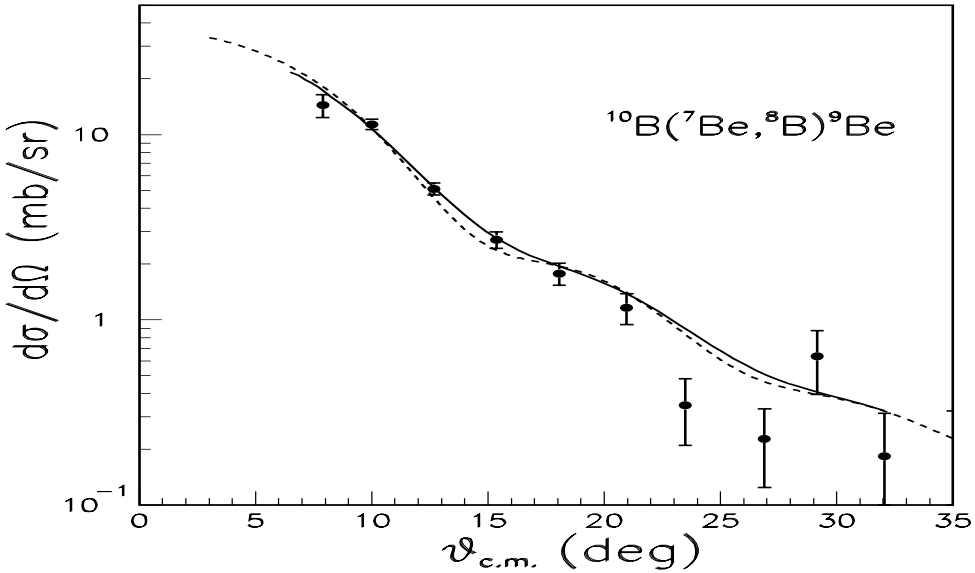


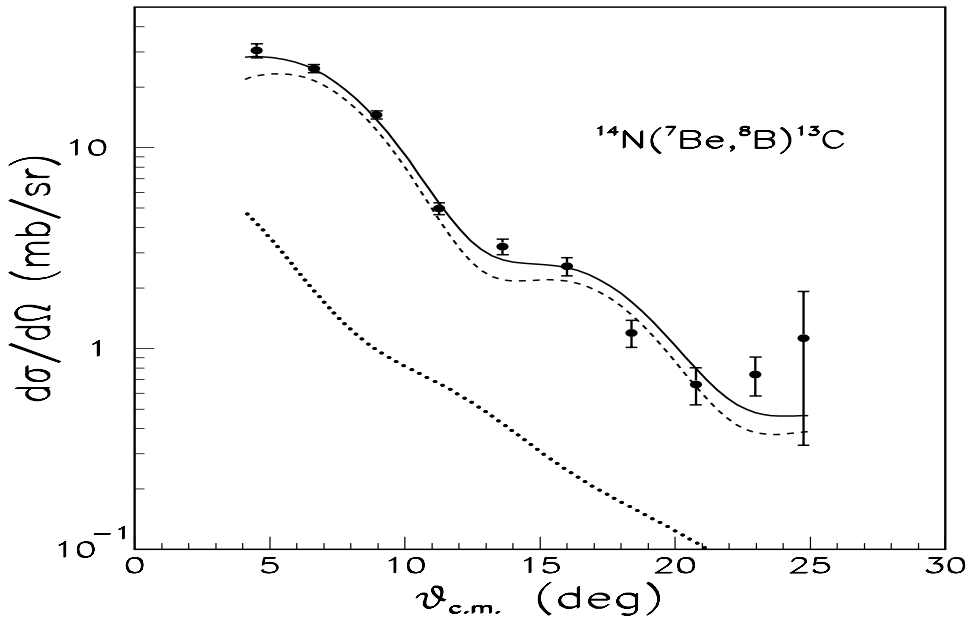


- ANC's ⇐ $^{10}B(^{7}Be,^{8}B)^{9}Be$ and $^{14}N(^{7}Be, ^{8}B)^{13}C$
- ^{7}Be beam – 84 MeV
- up to 80 kHz and > 99%
- E-ΔE Si telescopes
- JLM + folding model optical parameters from auxiliary study**
- elastic scattering check

[NOTE: these are not fits!!]

S factor for $^7Be(p, \gamma)^8B$



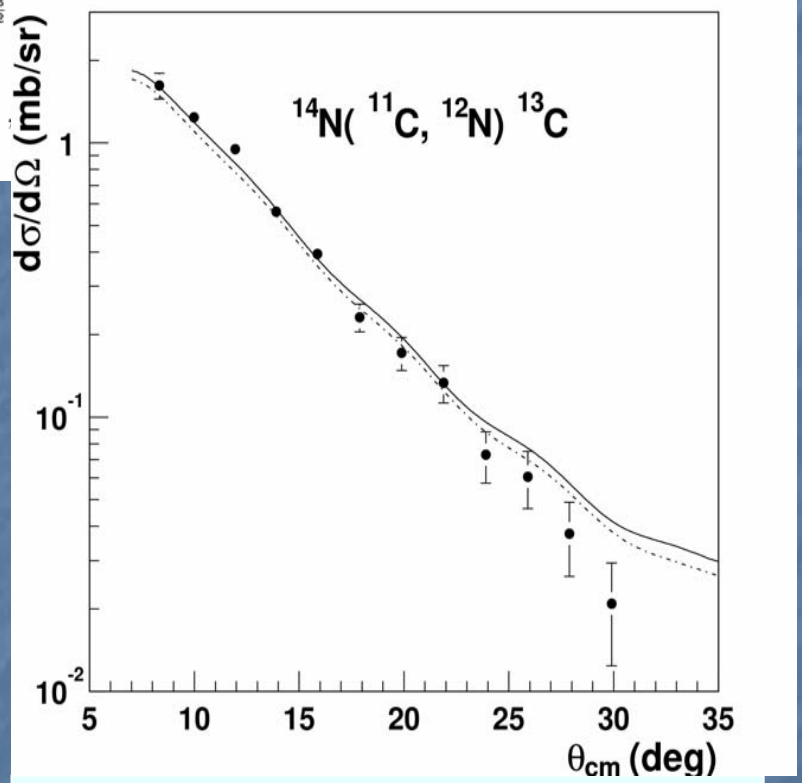


- transfer dσ/dΩ's
- dashed line gives dominant component
- solid line smoothed for angular acceptance
- Fits ⇒ **ANC's**
- $C^2(^{10}B) = .410 \pm .055\ fm^{-1}$
- $C^2(^{14}N) = .388 \pm .039\ fm^{-1}$
- $S_{17} = 17.3 \pm 1.8$ eVb

$^{14}N(^{11}C,^{12}N)^{13}C$ angular distribution

$$\frac{1}{(C^{12N}_{^{11}Cp1\frac{1}{2}})^2} = \frac{1}{\sigma_{exp}}\left(\left(\frac{C^{14N}_{^{13}Cp1\frac{1}{2}}}{b^{14N}_{^{13}Cp1\frac{1}{2}}\, b^{12N}_{^{11}Cp1\frac{1}{2}}}\right)^2 \sigma^{DW}_{1\frac{1}{2}1\frac{1}{2}} + \left(\frac{C^{14N}_{^{13}Cp1\frac{3}{2}}}{b^{14N}_{^{13}Cp1\frac{1}{2}}\, b^{12N}_{^{11}Cp1\frac{3}{2}}}\right)^2 \sigma^{DW}_{1\frac{1}{2}1\frac{3}{2}} + \mathcal{S}\left(\frac{C^{14N}_{^{13}Cp1\frac{1}{2}}}{b^{14N}_{^{13}Cp1\frac{3}{2}}\, b^{12N}_{^{11}Cp1\frac{1}{2}}}\right)^2 \sigma^{DW}_{1\frac{3}{2}1\frac{1}{2}} + \mathcal{S}\left(\frac{C^{14N}_{^{13}Cp1\frac{3}{2}}}{b^{14N}_{^{13}Cp1\frac{3}{2}}\, b^{12N}_{^{11}Cp1\frac{3}{2}}}\right)^2 \sigma^{DW}_{1\frac{3}{2}1\frac{3}{2}}\right)$$

Normalization(beam&target)	6.5%
Optical Model	8%
ANCs of ^{14}N	6.4%
Statistical Error	3%
MC parameters	2%
Total	12.7%

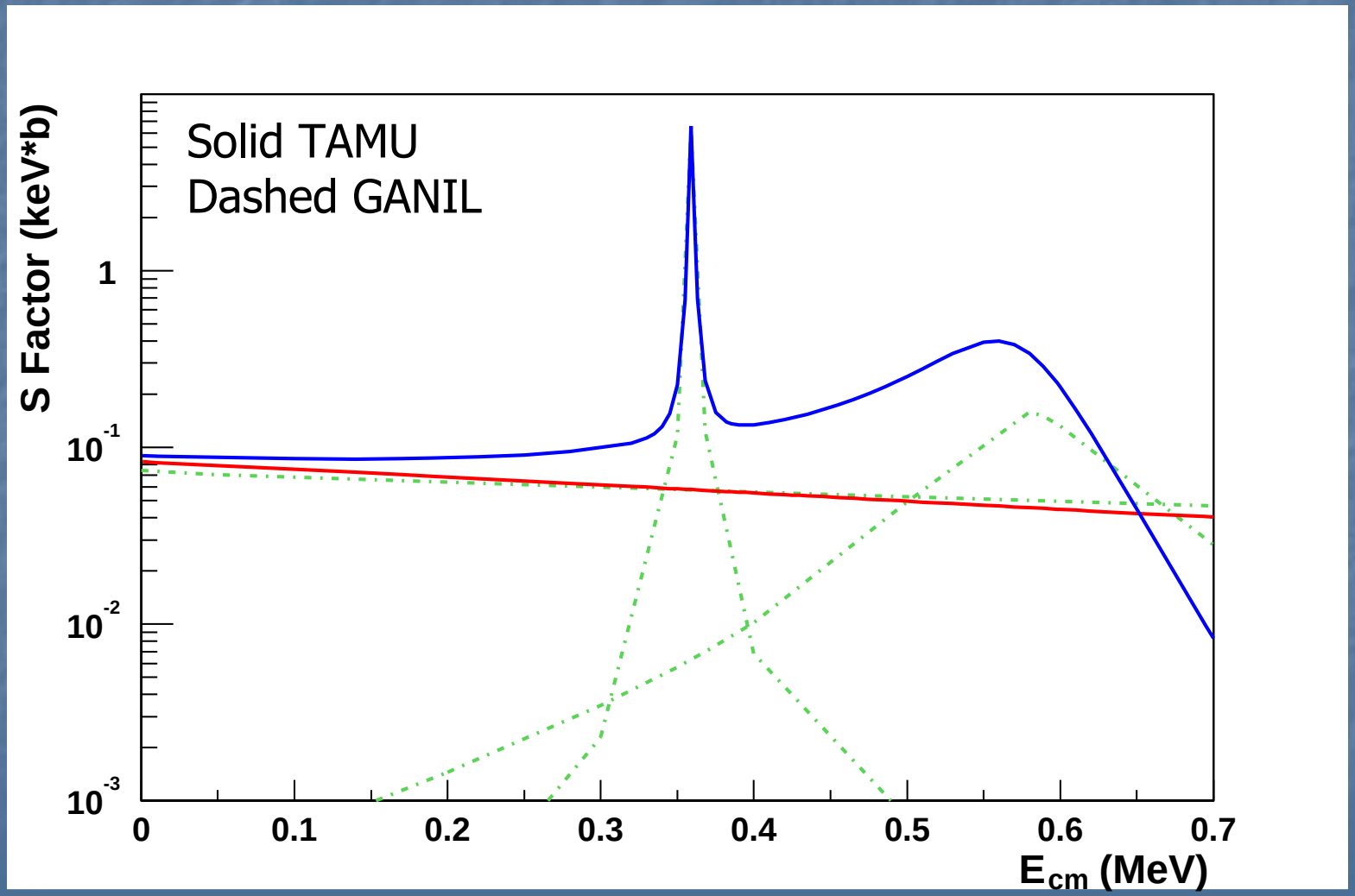


$$C^2_{1,1/2} = 1.4 \pm 0.2 \pm 0.07 \qquad fm^{-1}$$

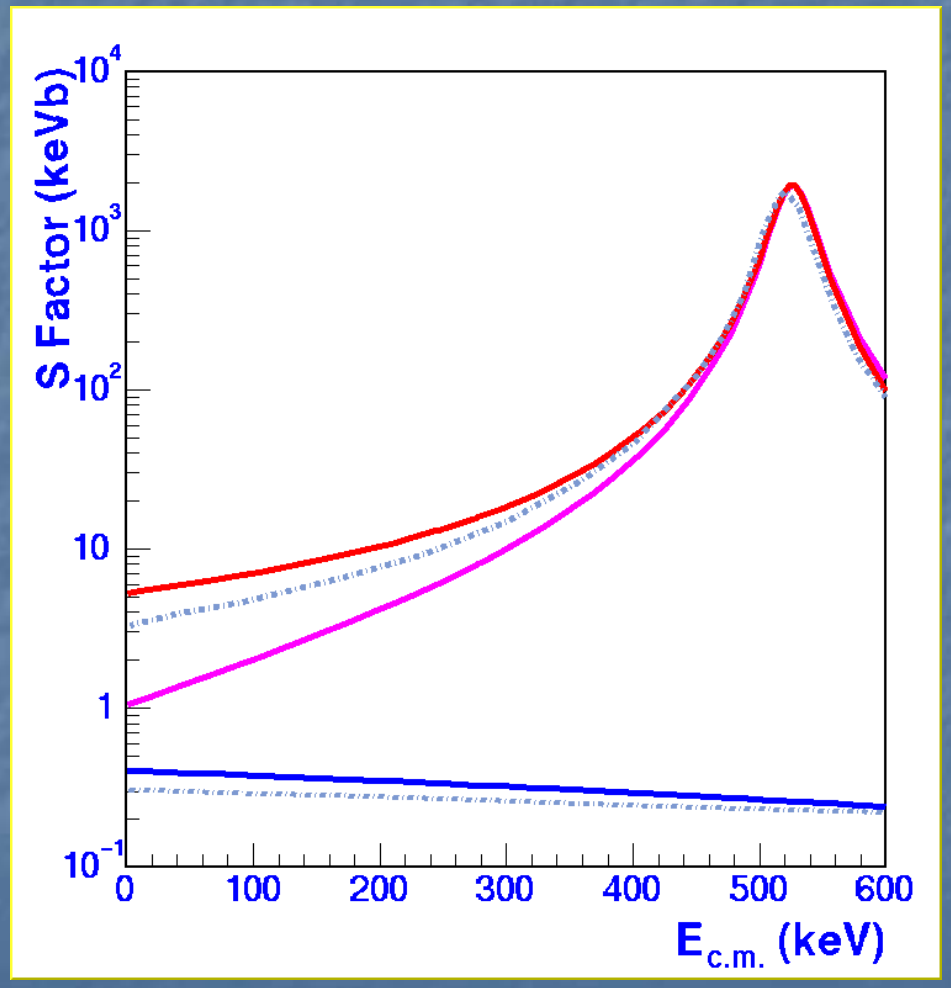
$$C^2_{1,3/2} = 0.33 \pm 0.05 \pm 0.05 \qquad fm^{-1}$$

$$s = \frac{C^2_{1,3/2}}{C^2_{1,1/2}} = \frac{0.17}{0.71} \qquad s \sim 20\%$$

S factor for $^{11}C(p,\gamma)^{12}N$



S Factor for $^{13}N(p,\gamma)^{14}O$



For Gamow peak at T_9=0.1,

- DC/Decrock_dc = 1.4
- Constructive/Decrock_tot =1.4
- Constructive/Destructive =4.0 (expected constructive interference for lower energy tail, useful to check)

ANC's measured by our group **stable beams** (mirror symmetry)

- **^{7}Be** + ***p*** $\leftrightarrow$ **^{8}B** [^{13}C(^{7}Li,^{8}Li)^{12}C]
- **^{22}Mg** + ***p*** $\leftrightarrow$ **^{23}Al** [^{13}C(^{22}Ne,^{23}Ne)^{12}C]
  [underway now]

ANC's for $^7Be + p \rightarrow {}^8B$ from mirror reaction $^{13}C(^7Li,^8Li)^{12}C$

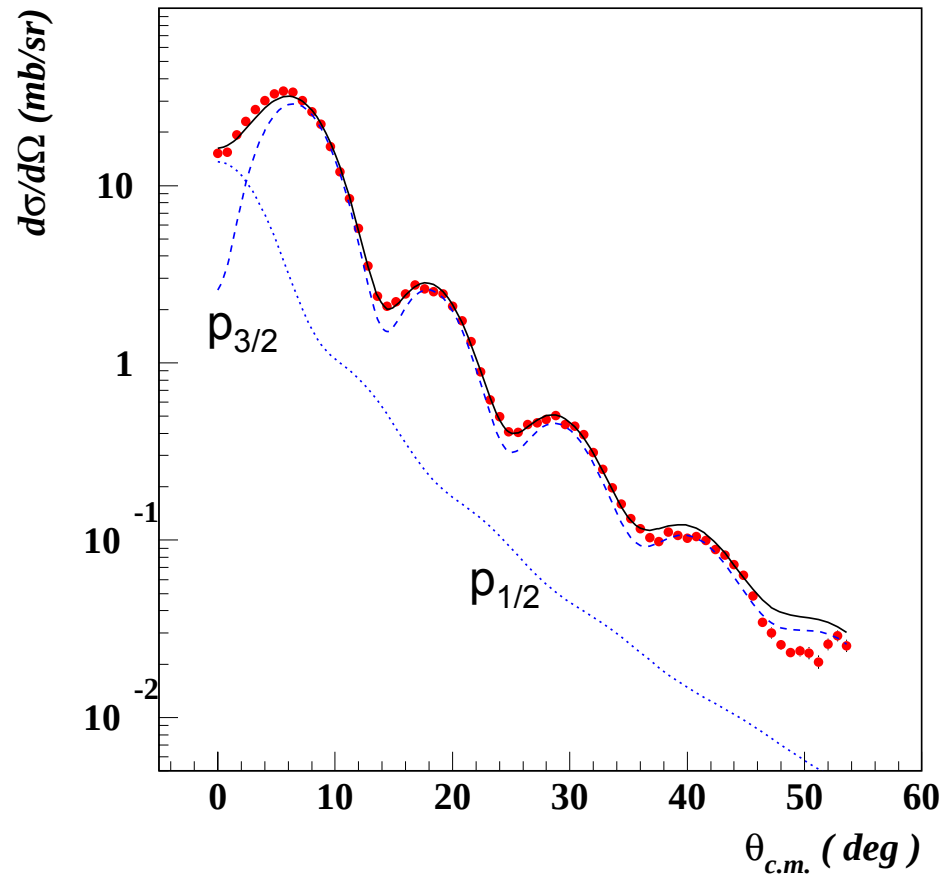
- separate $p_{1/2}$ and $p_{3/2}$
- Fits $\Rightarrow$ **ANC's**

$^7Li + n \rightarrow {}^8Li$:

- $C^2(p_{3/2}) = .384 \pm .038\ fm^{-1}$
- $C^2(p_{1/2}) = .048 \pm .006\ fm^{-1}$

$\Rightarrow$ $^7Be + p \rightarrow {}^8B$:

- $C^2(p_{3/2}) = .405 \pm .041\ fm^{-1}$
- $C^2(p_{1/2}) = .050 \pm .006\ fm^{-1}$



$S_{17}(0) = 17.6 \pm 1.7$ eVb

ANC's from Breakup

[Thanks to Jeff T. for Introduction!]

- breakup of **loosely bound nuclei**
  (many measurements available)
- BIG $d\sigma/d\Omega \Rightarrow$ can use radioactive beams
- tagging $\Rightarrow$ works with mixed beams
- measure $d\sigma/d\Omega$, parallel momentum dist.
- need good calculations

[Credit to F. Carstoiu and L. Trache]

An Extended Glauber Model approach

eikonal approximation - correction terms through second order

realistic interaction potentials:

- proton-target ⇒ JLM interaction
- core-target ⇒ renormalized double folding model using Hartree-Fock density distributions with JLM vary N_V from 0.37 to 0.8, N_W from 0.9 to 1.1

Coulomb interaction – includes both dipole and quadrupole

three components included in calculation:

stripping, diffraction dissociation and Coulomb interaction (E1 and E2)

$$\sigma_{sp} = \int_0^\infty 2\pi b db \left(P_{str}(b) + P_{diff}(b)\right) + \sigma_{Coul}$$

An Extended Glauber Model approach

usually assume structure: $$\Psi_{J^\pi} = \sum S^{1/2}(c,nlj)\left[\Phi_c^\pi \otimes \varphi_{sp}(nlj)\right]^{J^\pi}$$

then calculate: $$\sigma_{-1p} = \sum S(c,nlj)\sigma_{sp}(nlj) = \sum C_j^2 \frac{\sigma_j}{b_j^2}$$

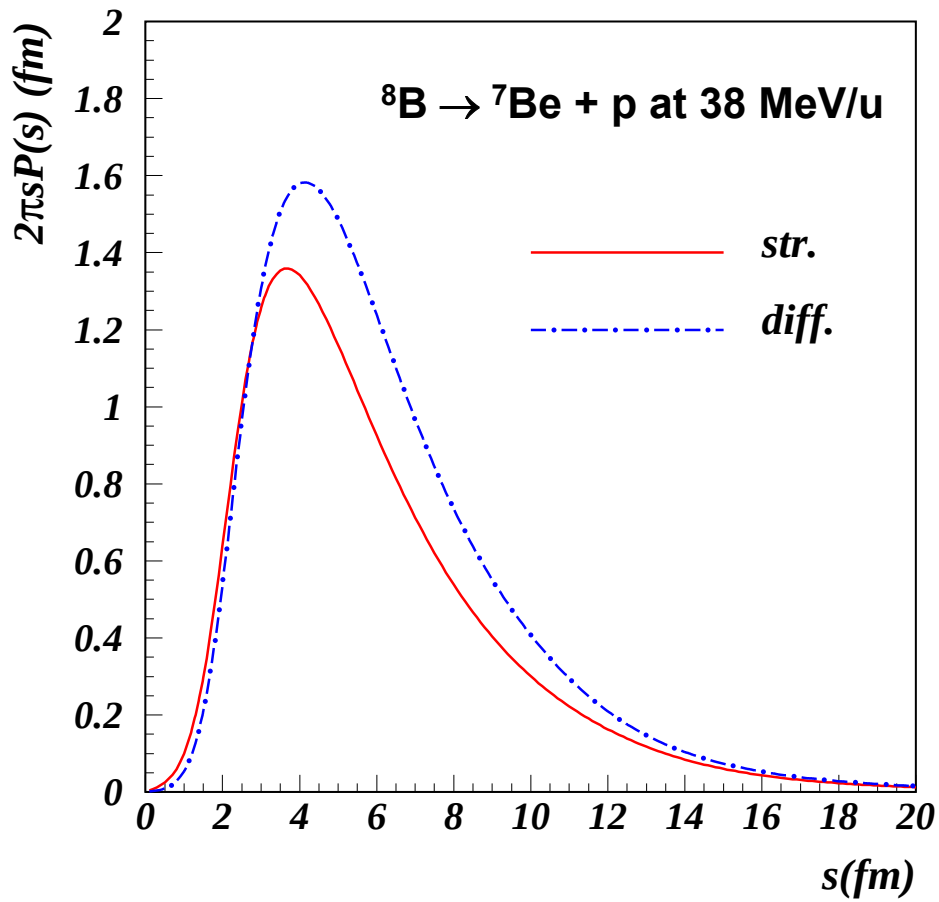
if reaction is peripheral, extract **ANC** for **^{8}B** via

$$\sigma_{-1p} = (S_{p_{3/2}} + S_{p_{1/2}})\sigma_{sp}(p_j) = (C^2_{p_{3/2}} + C^2_{p_{1/2}})\frac{\sigma_{sp}}{b_p^2}$$

Is process peripheral at E ≥ 30 MeV/u??

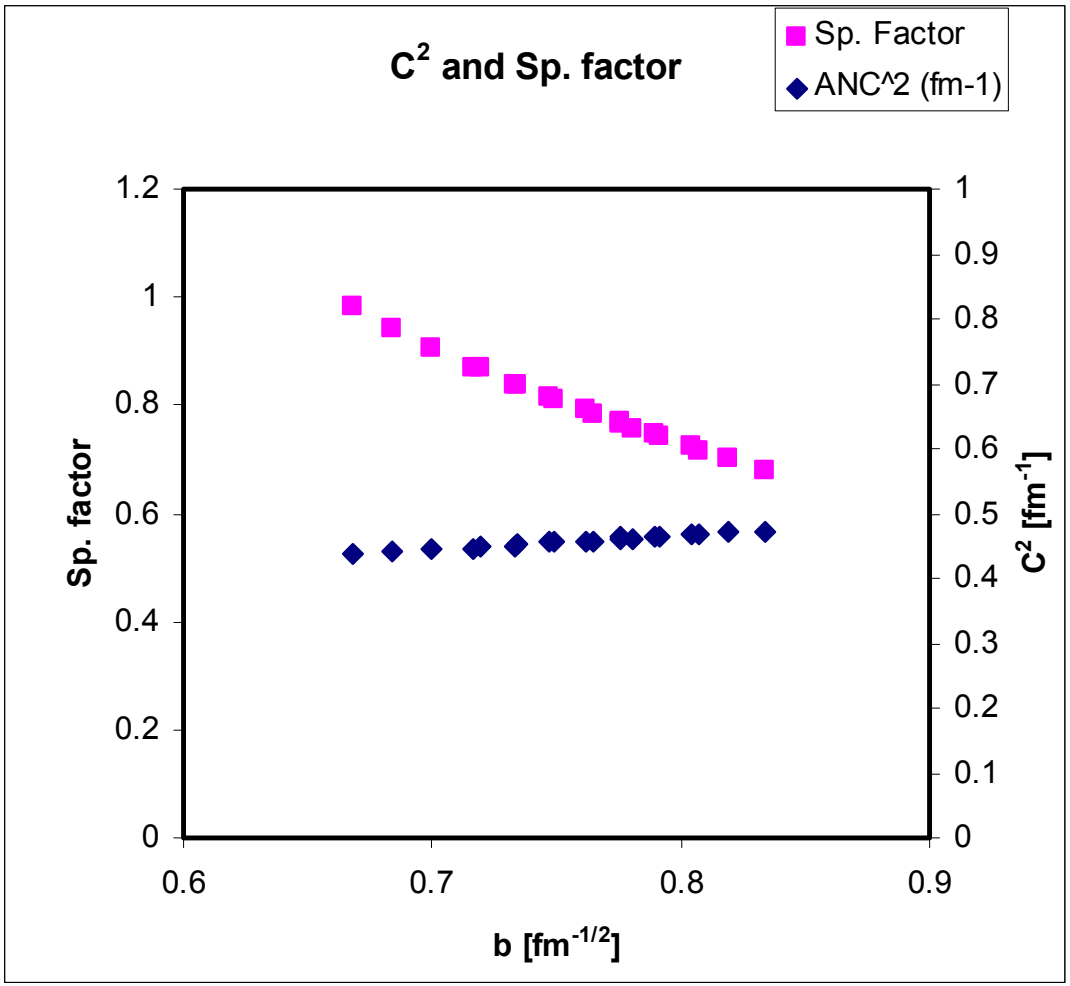
Is ^{8}B breakup peripheral?

- Si target
- contributions from stripping and diffraction vs. impact parameter peaked in surface region

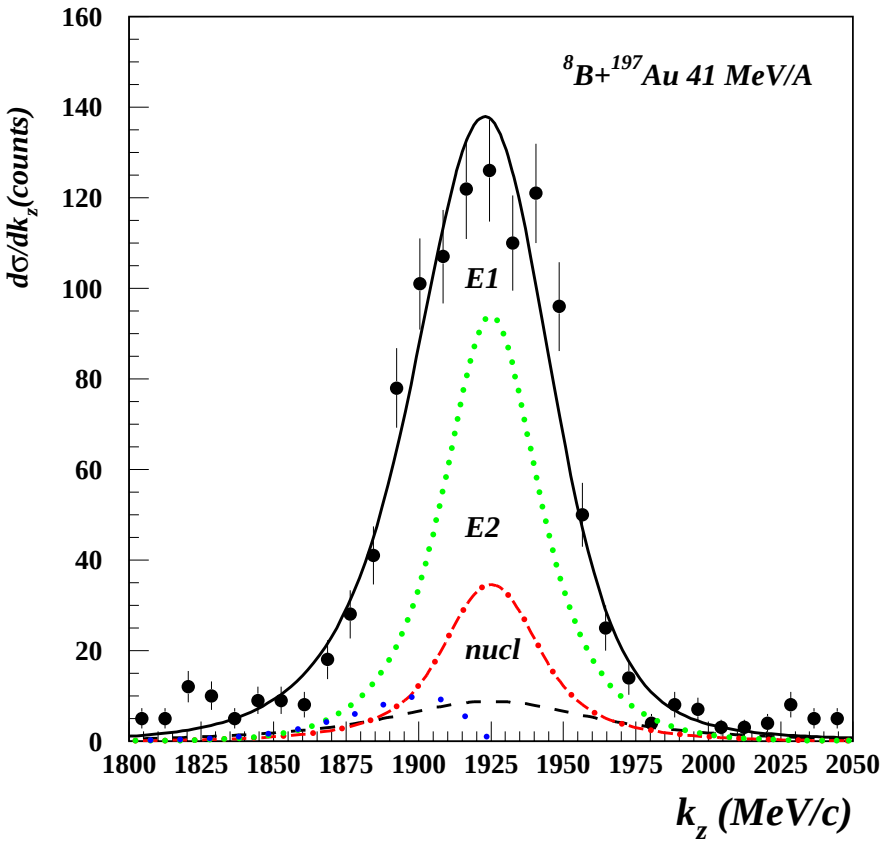


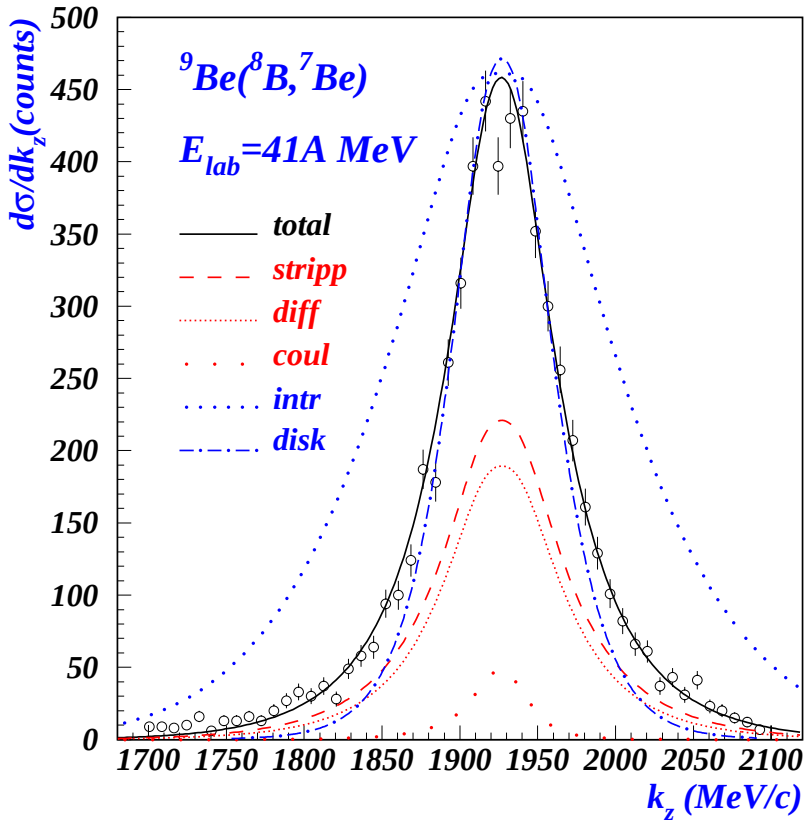
Is ^{8}B breakup peripheral?

- $C^2 \approx$ constant vs. s.p. well parameters, but S is not



Parallel momentum distributions

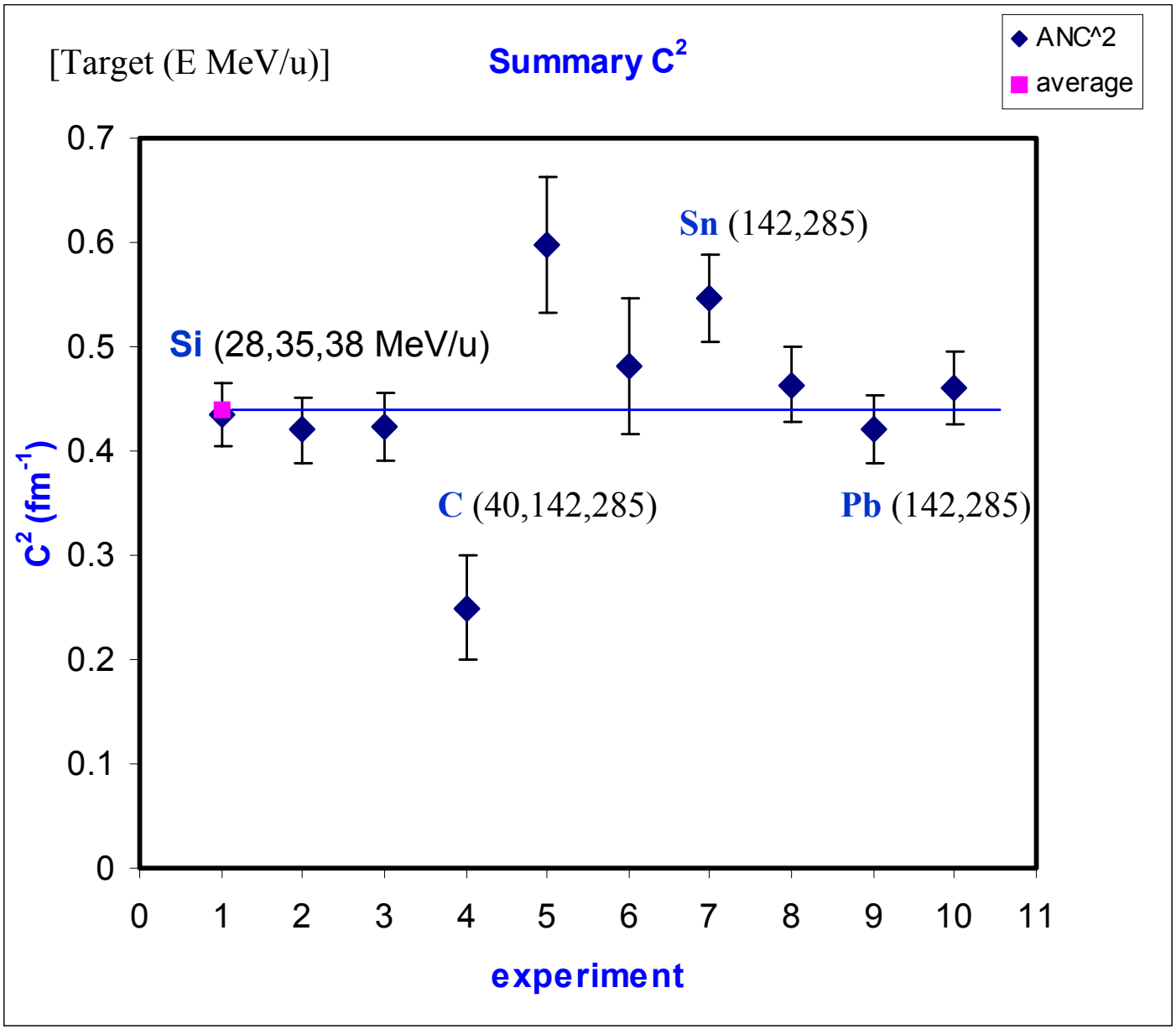




Glauber model calculation:

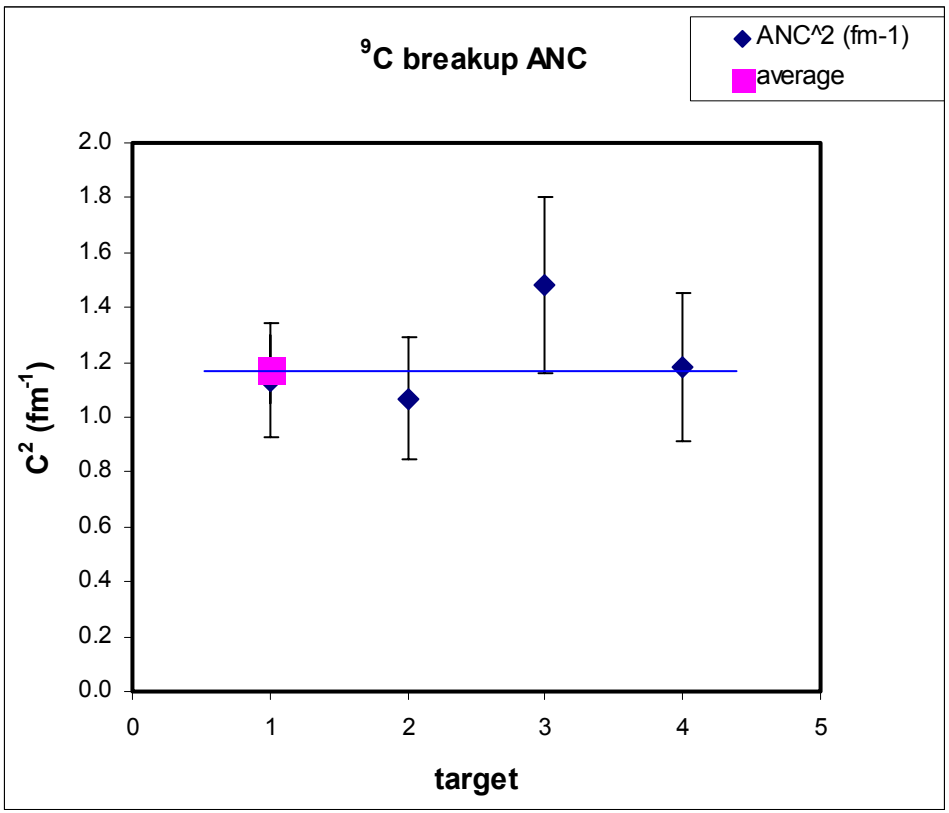
ANC used to get asymptotic normalization for wave function

ANCs from ^{8}B breakup



^{9}C breakup and ^{8}B(p,γ)^{9}C

- ^{8}B(p,γ)^{9}C($\beta^+\nu$)^{9}B(p)^{8}Be(α)^{4}He (pp IV)
- ^{8}B(p,γ)^{9}C(α,p)^{12}N (rap I)



[Data from Beaumel et al.]

- $C^2 = 1.22 \pm 0.13$ fm^{-1}
- $S_{18}(0) = 46 \pm 6$ eVb
- Wiescher $\Rightarrow$ 210 eVb
- d(^{8}B,^{9}C)n

  $C^2 = 1.2 \pm 0.34$ fm^{-1}

  (14.4 MeV/u)

Summary/Outlook

- ANCs $\Rightarrow$ tool for nuclear astrophysics
- Measure ANCs via transfer reactions
- Measure ANCs via breakup reactions
- Extend to *s*-*d* shell nuclei
- Applications with radioactive beams
- Utilize mirror symmetry

Collaborators

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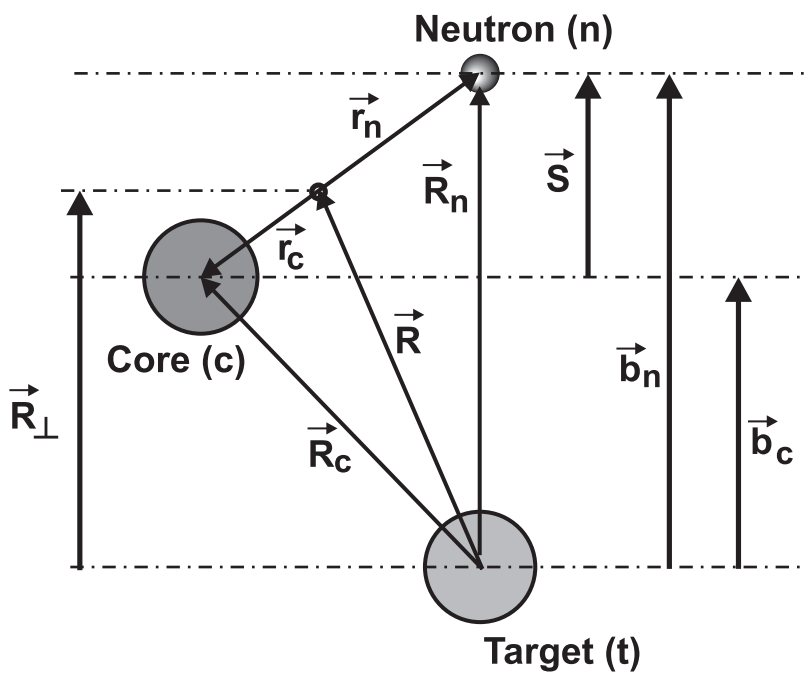
- **INP** (Czech Republic): P. Bem, V. Burjan, V. Kroha, J. Novak, S. Piskor, E. Simeckova, J. Vincour
- **IAP** (Romania): F. Carstoiu

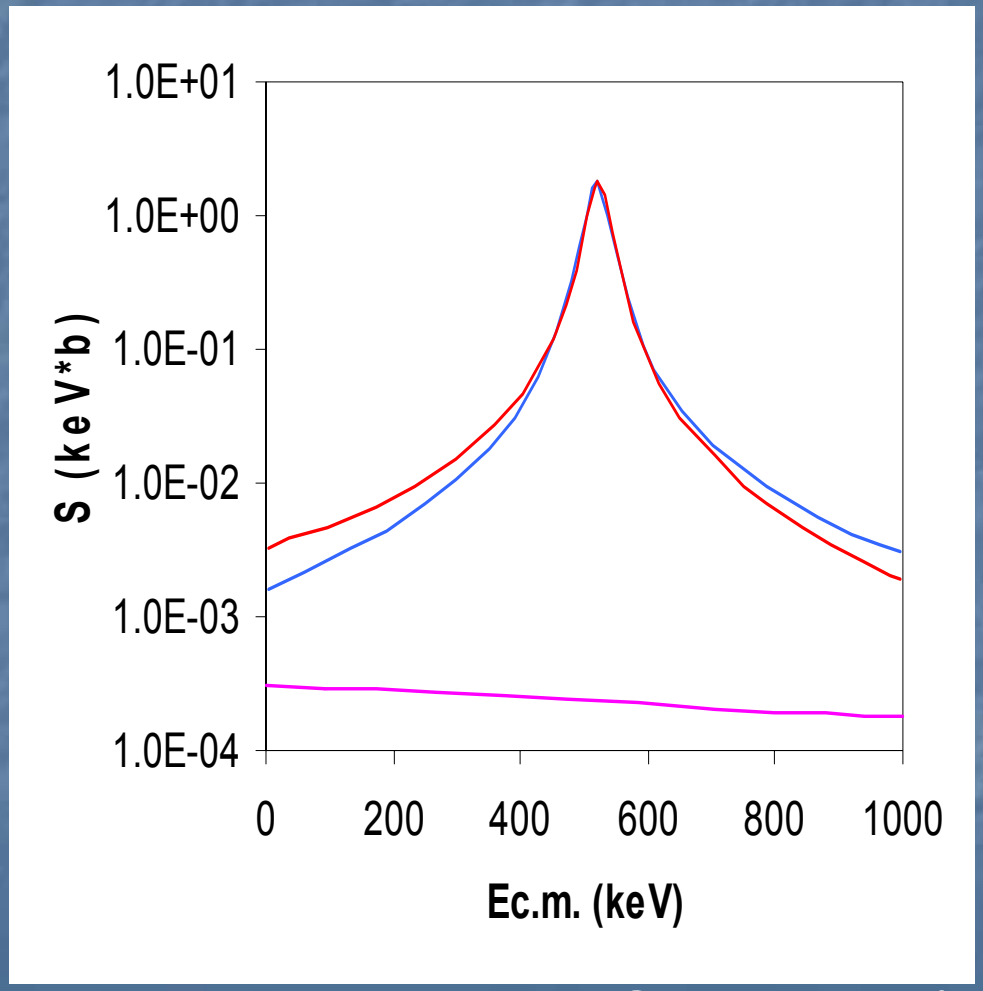
Glauber Model geometry



impact parameter $b = R_\perp$

Target-core potentials - double folding using JLM interaction

$^{13}N(p,\gamma)^{14}O$



$$\Gamma_{tot} = 37.3 \pm 0.9 keV$$

$$(^{14}N(^{3}He,t)^{14}O, p(^{13}N,p)^{13}N)$$

$$\Gamma_{\gamma} = 3.36 \pm 0.72 eV$$

$$(^{13}N(p,\gamma)^{14}O, ^{12}C(^{3}He,n\gamma)^{14}O,$$

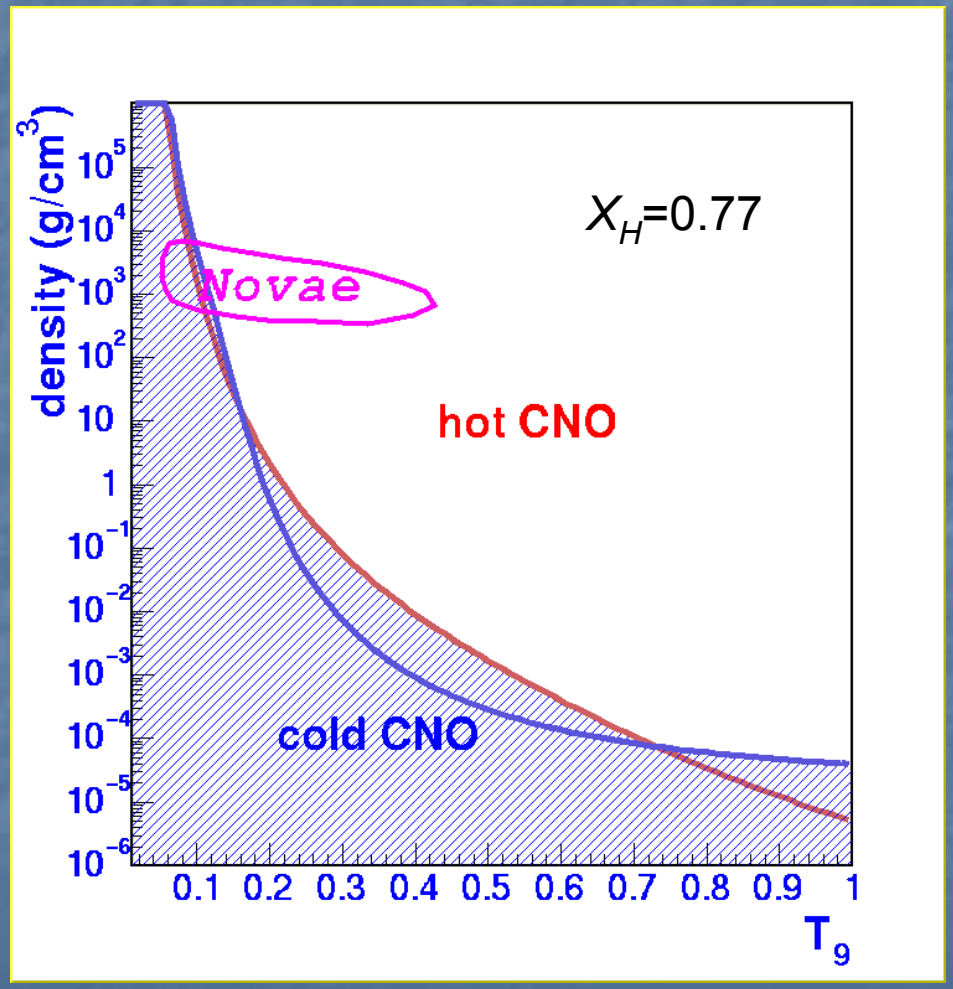
$$^{1}H(^{14}N,^{14}O)n, ^{208}Pb(^{14}O,^{13}Np)^{208}Pb)$$

$$S = 0.90(0.23)$$

$$(^{13}N(d,n)^{14}O)$$

D.Decrock et al., PRC48, 2057(1993)

Transition from CNO to HCNO



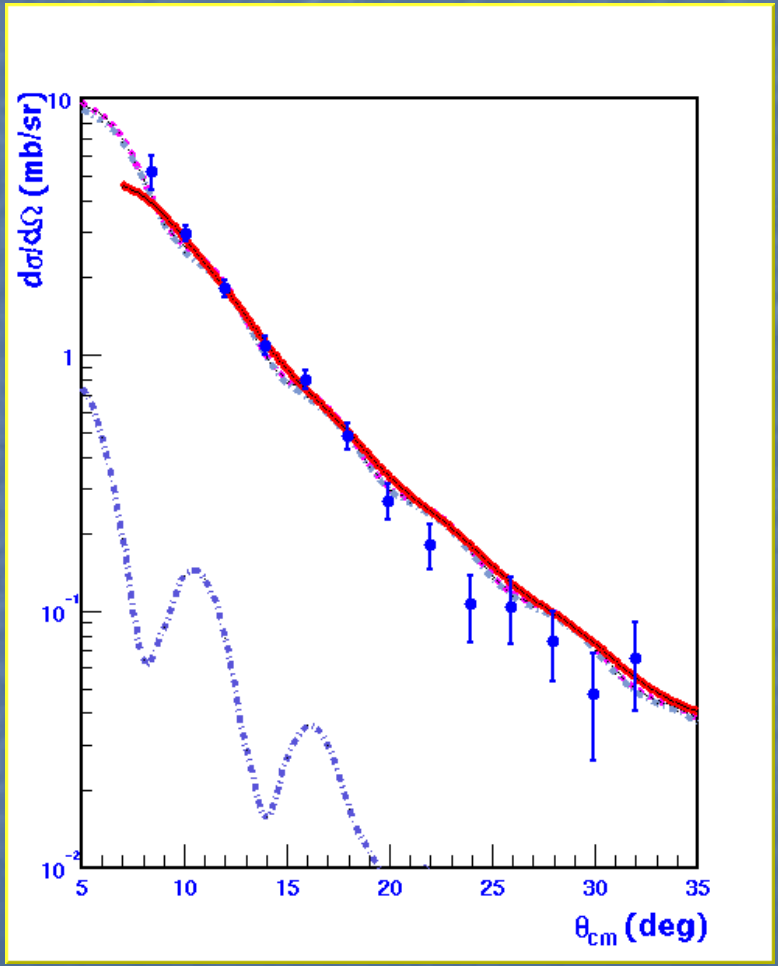
Crossover at $T_9 \approx 0.2$

- $^{13}N(p,\gamma)^{14}O$ vs β decay
- $^{14}N(p,\gamma)^{15}O$ vs β decay

For novae find that $^{14}N(p,\gamma)^{15}O$ slower than $^{13}N(p,\gamma)^{14}O$; $^{14}N(p,\gamma)^{15}O$ dictates energy production

$^{14}N(^{13}N,^{14}O)^{13}C$

(ANC for $^{14}N \rightarrow ^{13}C + p$)



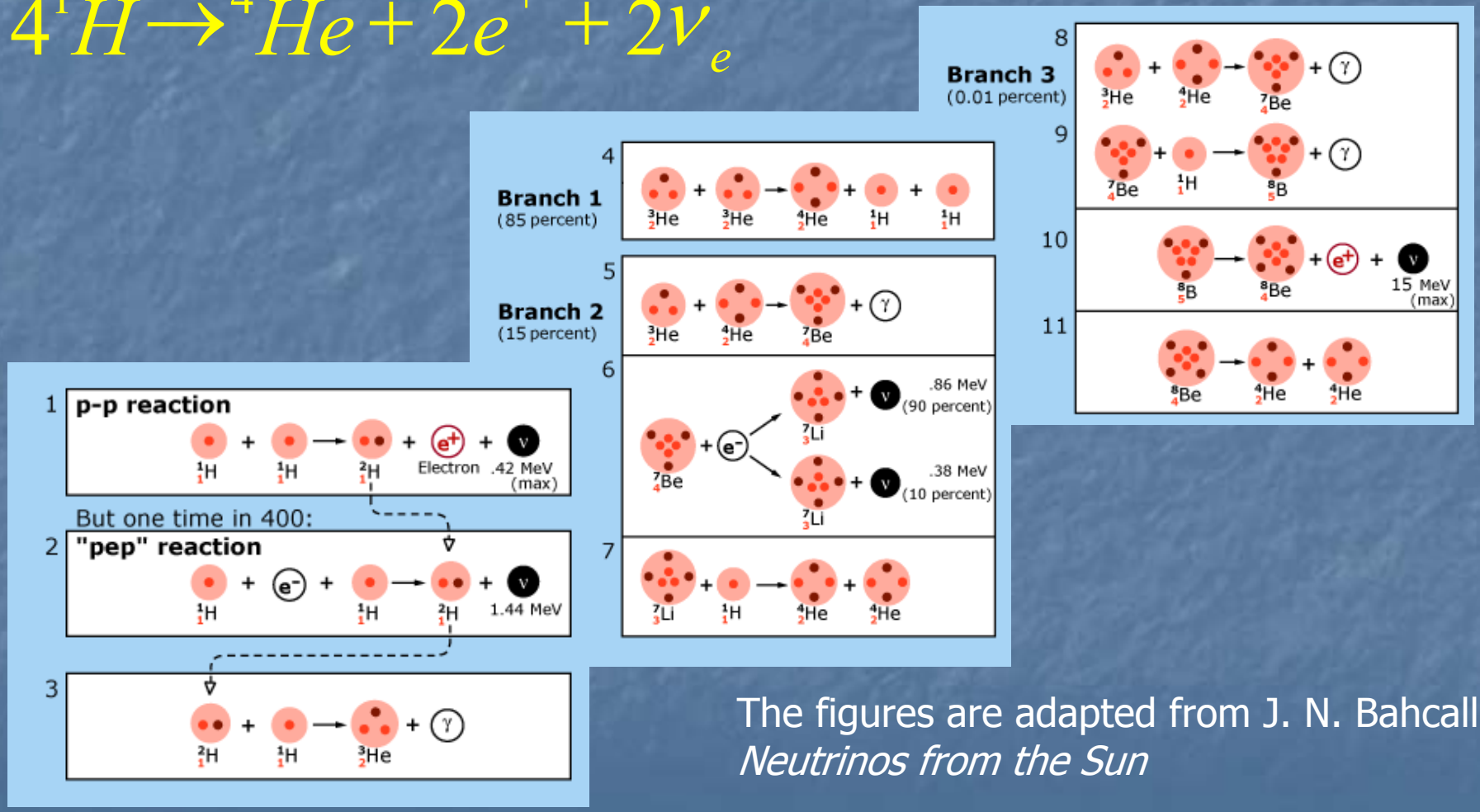
DWBA by FRESCO

$$\sigma_{\exp} = \left(C^{^{14}O}_{^{13}N\frac{1}{2}} \right)^2 \left(\left(\frac{C^{^{14}N}_{^{13}C\frac{3}{2}}}{b^{^{14}N}_{^{13}C\frac{3}{2}} b^{^{14}O}_{^{13}N\frac{1}{2}}} \right)^2 \sigma^{DW}_{\frac{1}{2}\frac{3}{2}} + \left(\frac{C^{^{14}N}_{^{13}C\frac{1}{2}}}{b^{^{14}N}_{^{13}C\frac{1}{2}} b^{^{14}O}_{^{13}N\frac{1}{2}}} \right)^2 \sigma^{DW}_{\frac{1}{2}\frac{1}{2}} \right)$$

C^2 = 29.0 ± 4.3 fm^{-1}

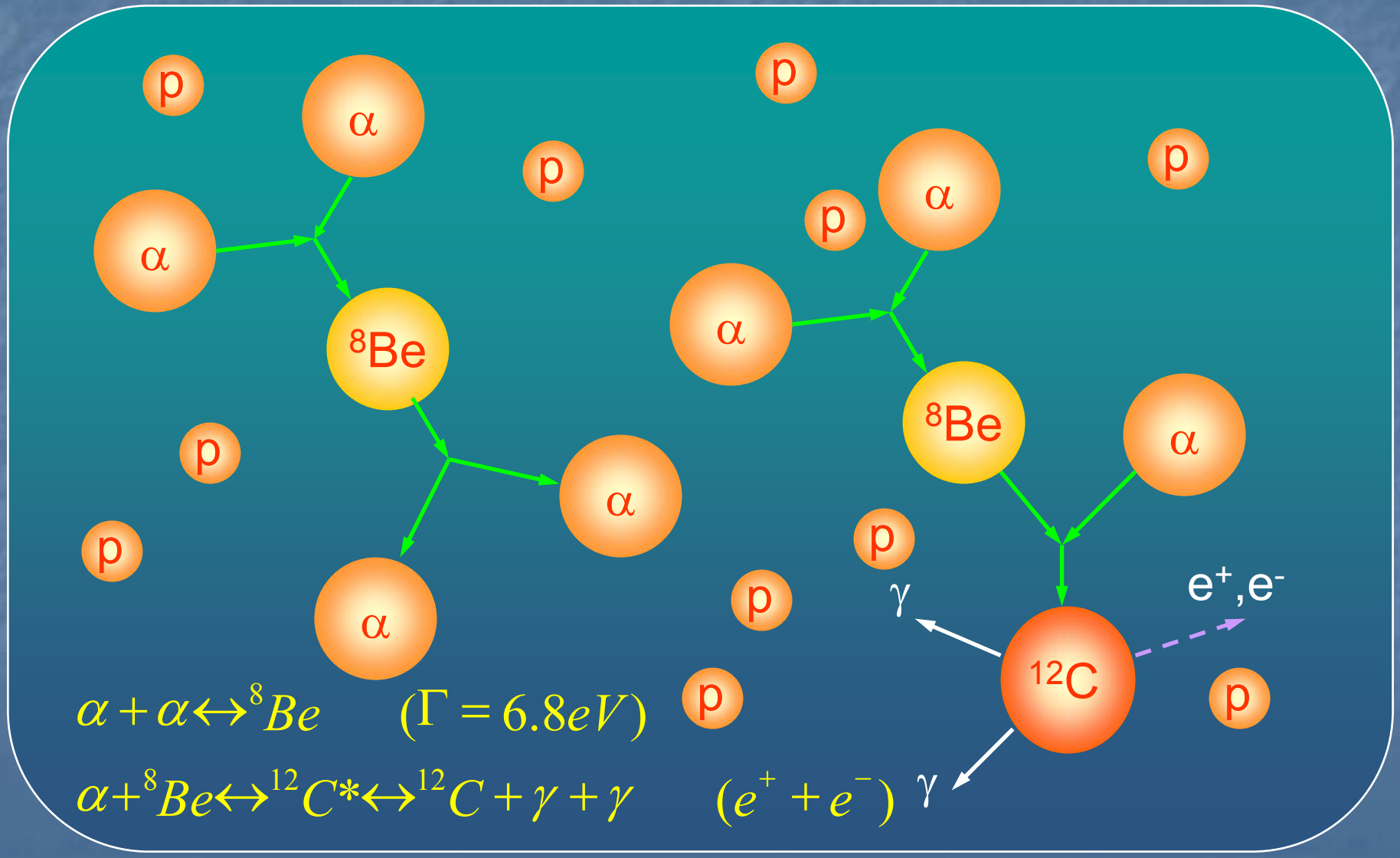
The p—p chain reaction

$$4\,{}^{1}H \rightarrow {}^{4}He + 2e^{+} + 2\nu_{e}$$



The figures are adapted from J. N. Bahcall, *Neutrinos from the Sun*

Triple Alpha Process



$$\alpha + \alpha \leftrightarrow {}^8Be \qquad (\Gamma = 6.8eV)$$

$$\alpha + {}^8Be \leftrightarrow {}^{12}C^* \leftrightarrow {}^{12}C + \gamma + \gamma \qquad (e^+ + e^-)$$

S factor for $^{14}N(p, \gamma)^{15}O$

S factor dominated by direct capture to the subthreshold state

γ width is small $\Rightarrow$

Contribution from the **tail** of the subthreshold resonance: $3/2^+, E_R = -504\,\mathrm{keV}$ is negligible

New reaction rate at $.007 < T_9 < 0.1$ **lower** (up to 2) than NACRE.

(Impacts stellar luminosity at transition period to red giants and ages of globular clusters.)

Subthreshold Capture

p radiative capture:

$$M \propto \int_{r_o}^{\infty} dr\, r^L \varphi_{l_f}(r)\left(e^{-ikr} - Se^{ikr}\right)$$

near threshold – ***S*** matrix is:

$$S = e^{2i\delta} = e^{2i\delta_p} \frac{k^2 - k_0^2 - i\gamma(k)}{k^2 - k_0^2 + i\gamma(k)}$$

where:

$$\gamma = \mu\Gamma = P_l \frac{W^2_{-\eta,l+1/2}(2\kappa r_0)}{r_0}|C|^2$$

$$k_0^2 < 0 \ (bound\ state) - ik_0 = \kappa = \sqrt{2\mu\varepsilon}$$

$$k_0^2 > 0 \ (resonance\ state)$$